Convolutional Codes Definitions

Constraint length maximum number of bits in a single output stream that can be affected by any input bit

- There is no standard definition
- In Wicker it is defined as

$$K = 1 + \max_i m_i$$

- In Lin and Costello: Maximum number of encoder outputs that can be affected by a single information bit

$$K = n(m + 1)$$

  where m is the memory order, $m = \max_i m_i$

- In Forney: number of information bits that must be saved

$$K = \sum_i m_i$$

Total memory total number of memory elements

$$M = \sum_{i=0}^{k-1} m_i$$

Maximal memory order Length of the longest shift register

$$m = \max_i m_i = K - 1$$

Distance measures

minimum free distance

$$d_{\text{free}} \stackrel{\triangle}{=} \min\{d(\mathbf{v}', \mathbf{v}'') : \mathbf{u}' \neq \mathbf{u}''\}$$

d_{free} is the minimum distance between any two codewords in the code. Since a convolutional code is linear:

$$\begin{aligned} d_{\text{free}} &= \min\{w(\mathbf{v}' + \mathbf{v}'') : \mathbf{u}' \neq \mathbf{u}''\} \\ & \min\{w(\mathbf{v}) : \mathbf{u} \neq \mathbf{0}\} \end{aligned}$$

column distance function (CDF) Let

$$\begin{aligned} [\mathbf{v}]_i &= (v_0^{(1)} v_0^{(2)} \cdots v_0^{(n)}, v_1^{(1)} v_1^{(2)} \cdots v_1^{(n)}, \\ & \ldots, v_i^{(1)} v_i^{(2)} \cdots v_i^{(n)}) \end{aligned}$$

denote the ith truncation of the codeword $\mathbf{v}$ and

$$[\mathbf{u}]_i = (u_0^{(1)}u_0^{(2)}\cdots u_0^{(k)}, u_1^{(1)}u_1^{(2)}\cdots u_1^{(k)}, \ldots, u_i^{(1)}u_i^{(2)}\cdots u_i^{(k)})$$

denote the ith truncation of the information sequence $\mathbf{u}$. The column distance function of order i, d_i, is defined as

$$d_i \triangleq \min\{d([\mathbf{v}']_i, [\mathbf{v}'']_i) : [\mathbf{u}']_0 \neq [\mathbf{u}'']_0\}$$
$$\min\{w([\mathbf{v}]_i) : [\mathbf{u}]_0 \neq \mathbf{0}\}$$

So, d_i is the minimum-weight codeword over the first $(i+1)$ time units whose initial information block is nonzero.

minimum distance d_{min} of a (n, k) convolutional code with constraint length K is the CDF d_i evaluated at $i = K$.

Weight Enumerator

$$T(X,Y) = \sum_{i}^{\infty}\sum_{j=1}^{\infty} a_{ij}X^iY^j$$

where

- i is the weight of the output bit sequence
- j is the weight of the input bit sequence
- a_{ij} is the number of paths with input weight j and output weight i

path connects initial state to final state and does not go through any state twice

path gain product of the branch gains along the path

loop circuit that does not any state more than once

forward loop starts and stops in S_0 (same as a path).

A set of loops is nontouching if they do not share a common vertex

$K = \{K_1, K_2, \ldots\}$ is the set of all forward loops

$F = \{F_1, F_2, \ldots\}$ is the set of corresponding path gains

$L = \{L_1, L_2, \ldots\}$ is the set of all loops that do not contain S_0

$C = \{C_1, C_2, \ldots\}$ is the set of corresponding path gains

$$T(X,Y) = \frac{\sum_l F_l \Delta_l}{\Delta}$$

where Δ is the graph determinate

$$\begin{aligned}\Delta &= 1 - \sum_{L_l} C_l + \sum_{(L_l, L_m)} C_l C_m \\ &\quad - \sum_{(L_l, L_m, L_n)} C_l C_m C_n + \cdots\end{aligned}$$

where (L_l, L_m) are nontouching loop pairs, and where Δ_i is the cofactor of path K_i

$$\Delta_i = 1 - \sum_{(K_i, L_l)} C_l + \sum_{(K_i, L_l, L_m)} C_l C_m - \cdots$$