Formula sheet for EE/OSE 601 Midterm

$$E = \int_{-\infty} \infty |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty} \infty |X(\omega)|^2 d\omega$$

$$E = \sum_{k=-\infty}^{\infty} |f(k)|^2$$

$$\begin{aligned} P &= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \\ &= \frac{1}{T} \int_{t_0}^{t_0+T} |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |\alpha_n|^2 \text{ if } x(t) \text{ is periodic} \end{aligned}$$

$$P = \lim_{T\to\infty} \frac{1}{2k+1} \sum_{k=-\infty}^{\infty} |f(k)|^2$$

For Capacitors:

$$i(t) = C\frac{dv}{dt} \quad v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\lambda)d\lambda$$

For Inductors:

$$v(t) = L\frac{di}{dt} \quad i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\lambda)d\lambda$$

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda = h(t) * x(t)$$

If $f(t)$ and $h(t)$ are periodic with the same period:

$$g(t) = f(t) * h(t) = \frac{1}{T}\int_{0}^{T} f(\tau)h(t-\tau)d\tau$$

Correlation:

$$R_{fh}(t) = f(t) \star h^*(t) = \int_{-\infty}^{\infty} f(\tau)h^*(\tau - t)d\tau$$

$$R_f(t) = f(t) \star f^*(t) = \int_{-\infty}^{\infty} f(\tau)f^*(\tau - t)d\tau$$

Fourier Series:

$$f(t) = \sum_{n=-\infty}^{\infty} \alpha_n e^{jn\omega_0 t}$$

$$\alpha_n = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)e^{-jn\omega_0 t}dt = |\alpha_n| e^{j\phi_n}$$

$$f(t) = \alpha_0 + \sum_{n=1}^{\infty} 2|\alpha_n| \cos(n\omega_0 t + \phi_n)$$

$$f(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} [A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t)]$$

$$A_0 = 2\alpha_0 = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)dt$$

$$A_n = (\alpha_n + \alpha_n^\star) = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos(n\omega_0 t)dt$$

$$B_n = j(\alpha_n - \alpha_n^\star) = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin(n\omega_0 t)dt$$

Fourier Transform:

$$\mathcal{F}\{f(t)\} = F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

$$\mathcal{F}^{-1}\{F(\omega)\} = f(t) = \frac{1}{@\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$$

$$x_e(t) = 1/2[x(t) + x(-t)]$$

$$x_o(t) = 1/2[x(t) - x(-t)]$$

System Response:

$$H(\omega) = \mathcal{F}\{h(t)\}$$

$$G(\omega) = F(\omega)H(\omega)$$

where $f(t)$ is input signal and $g(t)$ is output signal.
Ideal (distortionless) Filter:

$$H(\omega) = H_0 e^{-j\omega t_0} \text{ where } H_0 \text{ is a constant}$$

Phase Filter:

$$H(\omega) = H_0 e^{-j\theta_n(\omega)}$$

Average value of a signal:

$$A_x = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)dt$$

$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

$$\cos\omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$\sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

$$\int_{t_1}^{t2} x(t)\delta(t-t_0)dt = \begin{cases} x(t_0) & t_1 < t_0 < t_2 \\ 0 & \text{otherwise} \end{cases}$$

Laplace Transform

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} f(t)e^{-st}dt$$

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{st}ds$$

Z Transform

$$Y(z) = \mathcal{Z}\{y(k)\} = \sum_{k=0}^{\infty} y(k)z^{-k}$$

$$y(n) = \sum_{\text{all poles } z_i \text{ inside } C} \text{residue of } X(z)\frac{z^n}{z} \text{ at } z = z_i$$