# CPE 633
# Chapter 4 – Fault-Tolerant Networks

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**UAH**

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## Introduction

- Interconnection networks are widely used today.
- Examples
  - ______________________________
  - __________________, processors with local memory executing parts of a ______________ __________________
  - __________________ – independent processors communicating
- The organization, or ___________, of the network may provide only _______________ between a given source and a given destination.
- Fault tolerance is achieved by having __________ _____ and/or ______________.

## 4.1 Measures of Resilience

### - Graph-Theoretical Measures

- ____________________________ – the number of nodes or links necessary to disconnect a network
- _______________________ – the rate of increase in network ________________ as nodes (_________________ and __________________) fail

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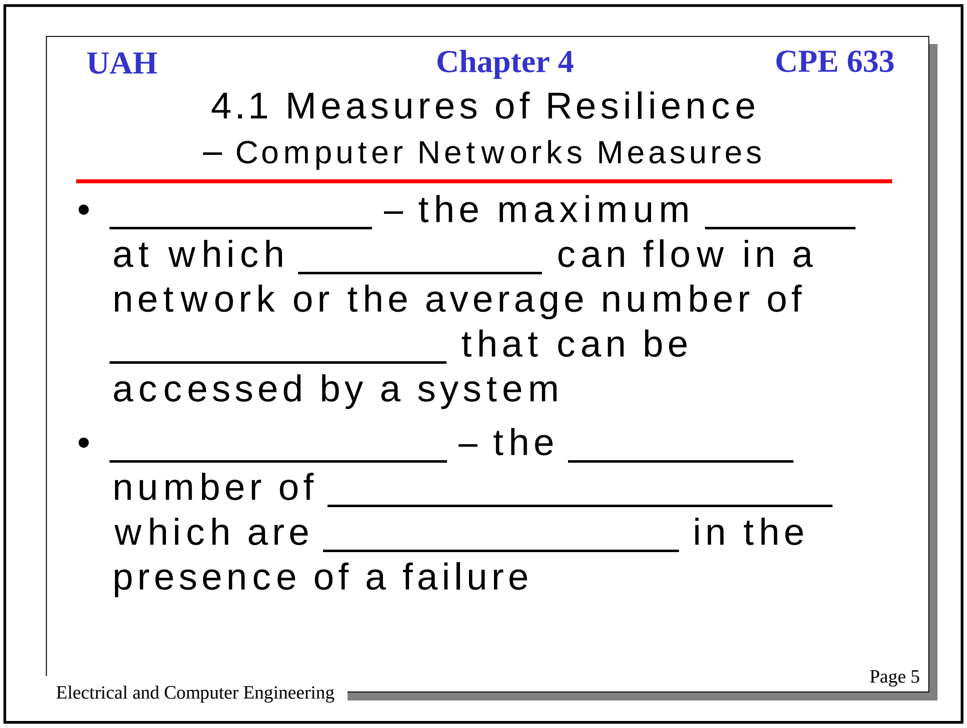
## 4.1 Measures of Resilience

### – Computer Networks Measures

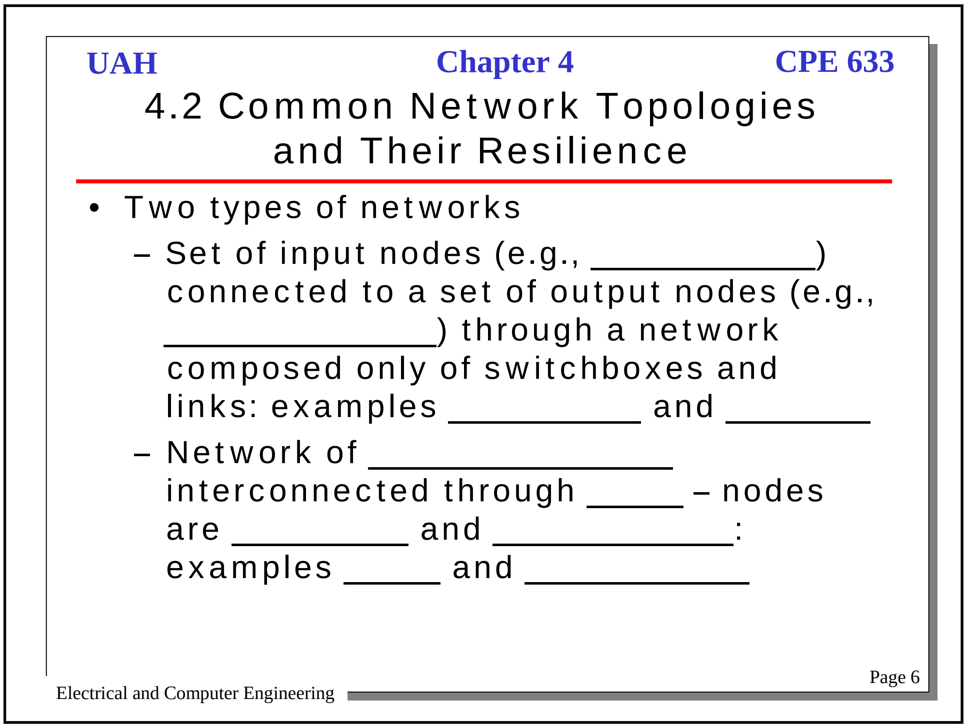
- _____________ – the network ___________ is the probability that all the nodes are ___________ and can _______________ with each other over the entire interval [0, t].
- If a specific __________________ pair is of special interest, we define the _______________ as the probability that an ___________________ has existed for this __________________________ during the ________ interval [0, t].
- If _________ is an integral component of the system’s management, _______________ can be used instead of _______________.



## 4.1 Measures of Resilience

### – Computer Networks Measures

- ______________ – the maximum ________ at which _____________ can flow in a network or the average number of __________________ that can be accessed by a system
- __________________ – the ____________ number of ___________________________ which are ___________________ in the presence of a failure



## 4.2 Common Network Topologies and Their Resilience

- Two types of networks
  - Set of input nodes (e.g., ____________) connected to a set of output nodes (e.g., _______________) through a network composed only of switchboxes and links: examples ___________ and ________
  - Network of _________________ interconnected through _____ – nodes are __________ and ______________: examples _____ and _____________

# 4.2.1 Multistage and Extra-Stage Networks

- Multistage networks are commonly used to connect a set of input nodes to a set of output nodes through either ____________ or ______________ links.
- These networks are typically built out of ____ ____________ that can have one of four settings
  - – ________
  - – ________
  - – ___________________
  - – ___________________


(a) S (b) C (c) UB (d) LB

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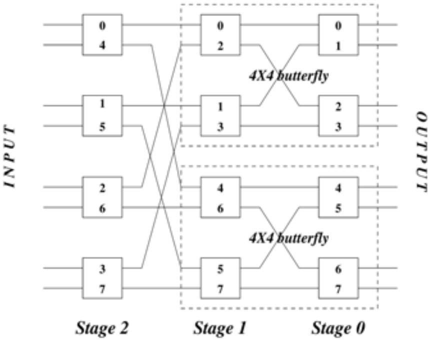
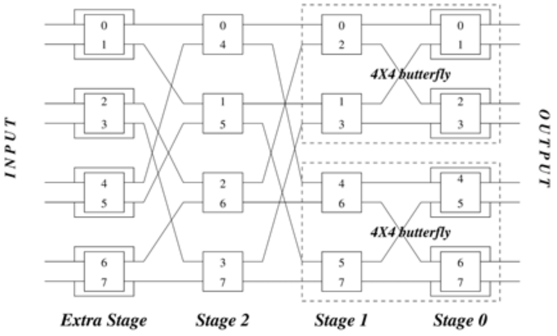
# 4.2.1 Multistage and Extra-Stage Networks – The Butterfly Network

- A well-known multistage network is the butterfly.
- k-stage network k ≥ 3
- A ________ butterfly connects $2^k$ outputs to $2^k$ outputs made up of __ _____ of _______________ each
- _______ is made up of two _____ plus an ____ ______
- As is, the butterfly is not ___________, only ____ ______ exists from every input to output



## 4.2.1 Multistage and Extra-Stage Networks – Modified Butterfly Network

- This network can remain connected despite the failure of up to one switchbox anywhere in the system



- The input nodes and the output nodes may be the same nodes

## 4.2.1 Multistage and Extra-Stage Networks – Butterfly Bandwidth Analysis

- Consider a ____________ interconnection network that connects N = $2^k$ ___________ to N = $2^k$ ________ units in a ________________ architecture
- Derive __________ – the expected number of ________ ________ from the processors that reach the ______________ – in the _____________________
- Assume ______ processor generates a __________ to a memory module with probability $p_r$ in ___________.
- ______ request is directed to _________ N memory modules with equal probability ____.
- The probability that a given processor generates a request to a __________ memory module i(i∈ {0,1,...,N-1}) is $p_r/N$ (assume requests are ____________)

## 4.2.1 Multistage and Extra-Stage Networks – Butterfly Bandwidth Analysis

- All __________ of a stage will carry a ______________ with the same probability, $p_r^{(i)}$, i=0, 1, …, k-1
- Calculate probabilities ____________, starting at the ______ (stage k-1) and working to the ________ (stage 0)
- The ______________ of each processor are divided _________between the two output lines of the switchbox on __________.
- Probability of a ________ on an ____________

$$p_r^{(k-1)} = \frac{p_r}{2} + \frac{p_r}{2} - \left(\frac{p_r}{2}\right)^2 = p_r - \frac{p_r^2}{4}$$

- That ___________ is the _____________ to the next stage.

$$p_r^{(i-1)} = p_r^{(i)} - \frac{(p_r^{(i)})^2}{4}$$

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## 4.2.1 Multistage and Extra-Stage Networks – Butterfly Bandwidth Analysis

- The bandwidth, BW = $Np_r^{(0)}$ is the expected number of requests making it to the memory end.
- Extend this analysis to include the ____________ of _______________. Assume that a __________ acts as an ______________.
- For any link, let $q_l$ be the probability that it is ______ and $p_l = 1 - q_l$ the probability that it is __________. Assume __________________ is incorporated into that of ____________________.
- The probability that a request at an input propagates to an output is $p_l p_r^{(i)}/2$

$$p_r^{(i-1)} = p_l p_r^{(i)} - \frac{(p_l p_r^{(i)})^2}{4}$$

## 4.2.1 Multistage and Extra-Stage Networks – Butterfly Connectability Analysis

- Network ____________ is the _____________ of connected processor-memory pairs.
- In a k-stage $2^k$ x $2^k$ butterfly network, there are ___ links and __ switchboxes to be ____________.
- $p_l$ and $p_s$ are the probabilities of ______ and _________ health, respectively. Link and switchbox failures are ______________.
- For a given ___________________ to be connected, all k+1 links and k switchboxes must be up ($p_\ell^{k+1}p_s^k$)
- For $2^{2k}$ input-output pairs, the expected number of pairs that are connected is given by $Q = 2^{2k} p_\ell^{k+1}p_s^k$

## 4.2.1 Multistage and Extra-Stage Networks – Butterfly Accessibility Analysis

- A processor is accessible if it is connected to _______ ____ memory.
- For accessibility calculation, assume that ________ fail.
- Start at the ____________, define $\phi(i)$ as the probability that at least _________________ exists from a __________ in stage i to the ______ end of the network.

☐ $\phi(0)$ is the probability that ________ line out of a switchbox at the output stage is ____________: this probability is $1 - q_l^2$

- Consider $\phi(i)$, i > 0. From ___________, we have two links. These two links never go to a ______ switchbox. A connection to the ________ exists through a link if ______________________ and the stage that it leads to is _________________. The probability of this is $p_l\phi(i-1)$.

## 4.2.1 Multistage and Extra-Stage Networks – Butterfly Accessibility Analysis

- The probability of a stage-i switchbox being ________________ from the output end is $(1 – p_l\phi(i-1))^2$
- The probability that a given processor can __________ the output end is given by $p_l\phi(k)$.
- Since there are $2^k$ processors, the expected number of __________________ that can connect to at least ____________ is $A_c = 2^k p_l\phi(k)$.
- Since the butterfly network is __________, this is also the expression for the ________________ of accessible ________________.

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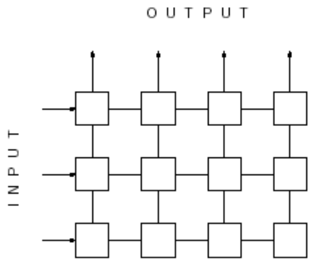
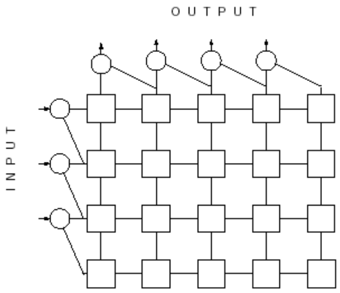
## 4.2.2 Crossbar Networks – Basic Operation

- Multistage Networks have _________. Consider processor __ communicating with memory __, processor __ is unable to communicate with memories __, __, and __ at the same time.
- A crossbar gives us more ________________________.
- For N inputs and M outputs, there is one _________ associated with each of the NM ______________ (i, j) switchbox.
- Each switchbox can
  - ____________________
  - ____________________
  - ____________________
  - ____________________
- A crossbar is not _____________
- Routing example: from 2 to 4



## 4.2.2 Crossbar Networks –
### Adding Fault-Tolerance

- Add an extra ______ and __________.
- Add extra ____________ from each _____ and ______ to ____ rows or columns.
- If any __________ become faulty, switch out ______ ________________ and use spares.



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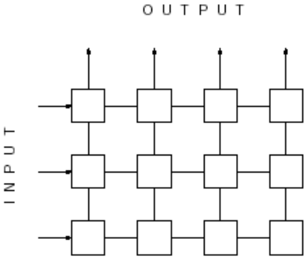
## 4.2.2 Crossbar Networks –
### Connectability of the Crossbar

- Assume ____________ are connected to the inputs and ______________ connected to the outputs.
- Counting from 1, for _______ to b connectable to ______, we have t go through a total of ____ links.
- The probability that all of them a _________ is $p_l^{i+j}$. Hence,

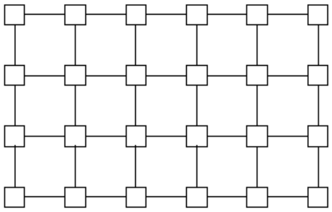
$$Q=\sum_{i=1}^{N}\sum_{j=1}^{M} p_l^{i+j} = p_l^2\,\frac{1-p_l^N}{1-p_l}\,\frac{1-p_l^M}{1-p_l}$$



## 4.2.3 Rectangular & Interstitial Mesh –

### Networks with no separate switchboxes

- A two-dimensional N x M _____________ mesh network is a ___________ in which all the nodes are _________ nodes.
- All nodes except the __________ nodes have 4 incident links (an important property for some algorithms).
- If any node fails, the _____ ________ (4 neighbors for internal nodes) is lost.
- $R_{mesh} = [R_{node}]^{NM}$



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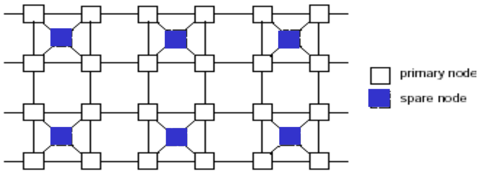
## 4.2.3 Rectangular & Interstitial Mesh–

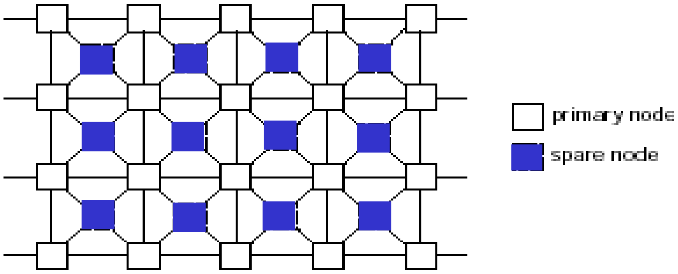
### Adding Redundancy: (1, 4) Scheme



- Add __________
- The sparing has two ___________
  - ______________________________
  - ______________________________________________
- Scheme shown is (1, 4)
  - Each ________ has a ______ spare
  - Each ______ is available to ________________
  - Overhead is 25 %

## 4.2.3 Rectangular & Interstitial Mesh–
### Adding Redundancy: (4, 4) Scheme

- Each _________ node has _________ nodes.
- Each _________ is available to _____________ nodes.
- Overhead is ((N-1)(M-1))/NM



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## 4.2.3 Rectangular & Interstitial Mesh–
### Reliability: (1, 4) Scheme

Let R(t) be the reliability of _______ primary or spare node, and let the mesh be of size _______ with both __ and __ _____ numbers.

The mesh contains ______ clusters of _________ nodes with _________ spare node.

The reliability of a _________, assuming that all links are __________, is

$$R_{cluster}(t) = R^5(t) + 5R^4(t)(1 - R(t))$$

and the reliability of the N x M ___________ mesh is

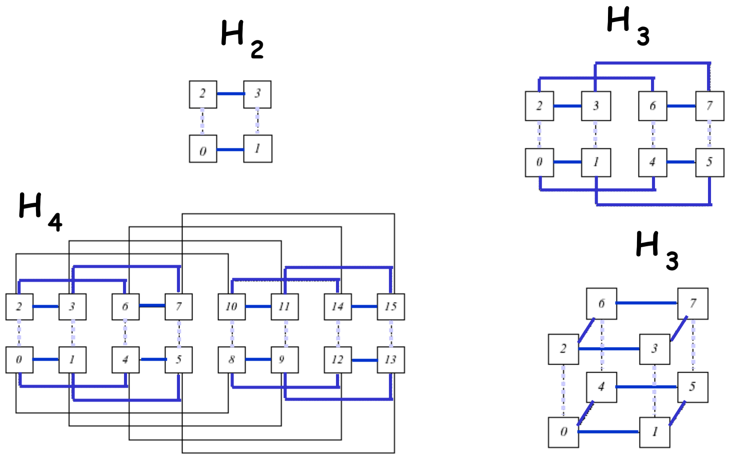
$$R_{IM}(t) = (R^5(t) + 5R^4(t)[1 - R(t)])^{NM/4}$$

compared to the original mesh $R_M(t) = R^{NM}(t)$

## 4.2.4 Hypercube Network – Definition

- A hypercube of __ dimensions, ___, consists of $2^n$ nodes and is constructed ____________.
- A _________________ hypercube, $H_0$, consists of just _________________.
- $H_n$ is constructed by taking _________ networks and connecting their ______________ nodes together. The ______ that are added to connect corresponding nodes in the two $H_{n-1}$ networks are called ___________ _______ edges.

## 4.2.4 Hypercube Network – Examples



# 4.2.4 Hypercube Network –
## Routing

- A node in a dimension-n hypercube has __ edges _________ upon it. ________________ from one node to another is quite simple if the nodes are __________ in the following way.
- When i and j are connected by a _______________ edge, the numbers of I and j differ in only the __________ position.
- Example: _____ and _____ differ in only bit position __, they must be connected by a ________________ edge.
- Routing: To go from 14 (1110) to 2 (0010) requires a dimension-2 edge and a dimension-3 edge. Two paths are possible: 1110→0110→0010, 1110→1010→0010
- The ____________ between the nodes is the same as the _______________________.
- For hypercubes of dimension ____, _____________ can be tolerated, because ________________ exist, but node failures can disrupt the operation of the network.

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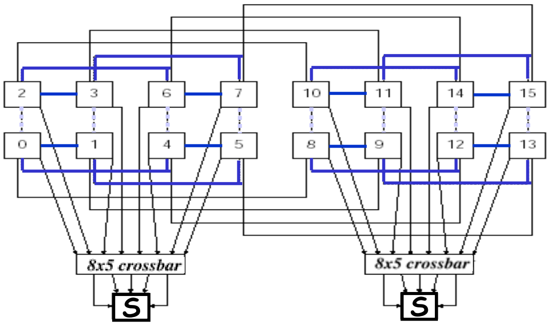
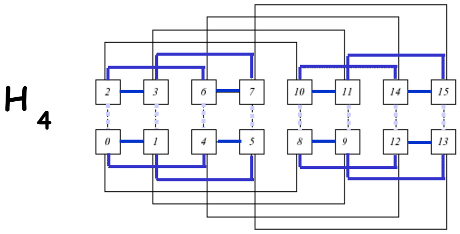
# 4.2.4 Hypercube Network –
## Adding Spare Nodes

- Sparing Option 1
  - Increase the number of communication ports of each node from n to (n + 1) and connect these extra ports through additional links to one or more spare nodes.



# 4.2.4 Hypercube Network – Adding Spare Nodes

- Sparing option 2
  - Duplicate the processor in a few selected nodes.
  - Each of these additional processors can serve as a spare, not only for the processor within the same node but also for any of the processors in the neighboring nodes.
  - Extra communication delay is required



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# 4.2.4 Hypercube Network – Reliability

- Assumption: nodes and links all fail ________________
- Reliability of $H_n$ is the _________ of
  - ______________ of $2^n$ nodes, and
  - Probability that __________ can communicate with __________________
- _______ evaluation of this probability difficult - every _______________ _______ connected by _______ _____
- Instead – we obtain a good _______ ______ on the reliability
- Exploiting _________ nature of hypercube – we add probabilities of three _________ _________ cases for which the _______ __ ____________
- This is a lower bound – there may be other cases where $H_n$ is connected

## 4.2.4 Hypercube Network –
### Reliability Defintions

- _______________ $H_n$ into two $H_{n-1}$ hypercubes, A and B, and the dimension-(n-1) links _____________ _______
- Case 1: Both A and B are operational and ___ _______ one dimension-(n-1) link is functional
- Case 2: One of A,B is operational and the _____ ___ ____ , and ____ dimension-(n-1) links are ___________
- Case 3: _____ ____ of A,B is operational, _________ ____ dimension-(n-1) link is _______ and is connected in the _____________ $H_{n-1}$ to a node that has ___ ______ ____ functional link to another node

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## 4.2.4 Hypercube Network –
### Reliability Case Probabilities

$q_c$ - probability of node failure , $q_l$- probability of link failure

$NR(H_n,q_l,q_c)$ – reliability of hypercube $H_n$

- Initial Assumption: Nodes are perfectly reliable ($q_c$=0)
- Case 1: Both A and B are operational and at least one dimension-(n-1) link is functional

$$\text{Prob}\{\text{Case 1}\} = [\text{NR}(H_{n-1}, q_\ell, 0)]^2 \left(1 - q_\ell^{2^{n-1}}\right)$$

- Case 2: One of A,B is operational and the other is not and all dimension-(n-1) links are functional

$$\text{Prob}\{\text{Case 2}\} = 2\text{NR}(H_{n-1}, q_\ell, 0)[1 - \text{NR}(H_{n-1}, q_\ell, 0)]\,(1 - q_\ell)^{2^{n-1}}$$

- Case 3: Only one of A,B is operational, exactly one dimension - (n-1) link is faulty and is connected in the nonoperational Hn-1 to a node that has at least one functional link to another node

$$\text{Prob}\{\text{Case 3}\} = 2\,\text{NR}(H_{n-1}, q_\ell, 0)[1 - \text{NR}(H_{n-1}, q_\ell, 0)] \times 2^{n-1} q_\ell (1 - q_\ell)^{2^{n-1}-1}(1 - q_\ell^{n-1})$$

## 4.2.4 Hypercube Network –
## Reliability Case Probabilities

$$\mathrm{NR}(H_n, q_\ell, 0) = \mathrm{Prob}\{\mathrm{Case}\ 1\} + \mathrm{Prob}\{\mathrm{Case}\ 2\} + \mathrm{Prob}\{\mathrm{Case}\ 3\}$$

- Initial case :
  - either _____________ of dimension __: ___ _____ and ___ _____

$$\mathrm{NR}(H_1, q_\ell, 0) = 1 - q_\ell$$

  - or, ____________ of dimension ___

$$\mathrm{NR}(H_2, q_\ell, 0) = (1 - q_\ell)^4 + 4q_\ell(1 - q_\ell)^3$$

- If the nodes are not perfect (qc ≠ 0)

$$\mathrm{NR}(H_n, q_\ell, q_c) = (1 - q_c)^{2^n}\,\mathrm{NR}(H_n, q_\ell, 0)$$

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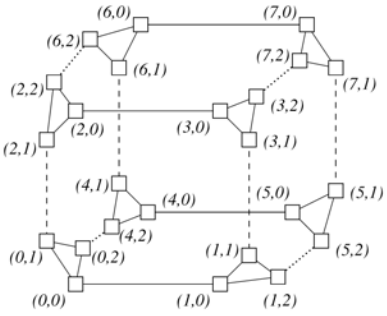
## 4.2.5 Cube-Connected Cycles Networks

- The hypercube has low __________ but high ______ and a new _____ ______ is necessary when __________ the size of a network.
- The Cube-Connected Cycles is an alternative that keeps the degree of a node _____ at three or less.
- A CCC network that corresponds to $H_3$ is shown.
- Each node in H3 is replaced by a _____ consisting of ______ _____.



## 4.2.5 Cube-Connected Cycles Networks – Labeling Scheme

- In general, each node of ______ __ in the hypercube $H_n$ is replaced by a ______ containing __ nodes where the degree of every node in the cycle is __.
- The resulting CCC(n,n) network has ____ nodes.
- By extending the ________ ______ of the hypercube, we can represent each node of the CCC by (i;j), where i (an n-bit binary number) is the label of the ____ __ ____ ______ that corresponds to the cycle and j(0 ≤ j ≤ n-1) is the position of the node _______ _____ ______.
- Two nodes, (I;j) and (I’;j’), are ______ ___ ___ _____ in the CCC if and only if either
  - i = i’ and j-j’ = ±1 mod n (__________________)
  - j = j’ and i differs from i’ in precisely the jth bit (_________________________________)

## 4.2.5 Cube-Connected Cycles Networks – Diameter and Reliability

- The _______ ________ of nodes in the CCC compared to the hypercube results in a _______ ___________.
- Instead of a diameter of a size n for the hypercube, the diameter of the CCC(n, n) is $2n + \left\lfloor \frac{n}{2} \right\rfloor - 2 \approx 2.5n$
- The _________ ___ __________ in the CCC is also more complicated than that in hypercubes. The fault tolerance of the CCC is, however, _______ because the failure of a ______ _____ in the CCC will only have an effect similar to that of a _____ _______ _____ in the hypercube.
- A _______ _____ expression for the reliability of the CCC _____ ____ ____ _____ derived.

## 4.2.6 Loop Networks

- The cycle(loop) _________ that is replicated in the CCC network can serve as an interconnection network with the desirable properties of a ________ _______ __________ and a _____ _____ _______.
- However, an _______ ____ with all its edges ______________ has a diameter of ___.
- Moreover, a ______________l loop network is not __________; a single ____ or ____ failure will disconnect the network.
- To _______ the diameter and ________ the fault tolerance of the loop network, extra links can be added (______).

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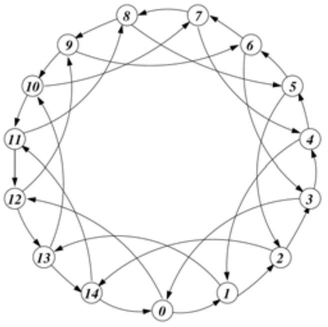
## 4.2.6 Loop Networks –
### Chordal Networks

- Example: 15-node chordal network with a skip distance of 3
- Node i (0 ≤ i ≤ n-1) has a ______ link to node (i + 1)mod n and a ________ link to node (i - s) mod n
- The ______ of every node in this ______ network is __ for any value of n.
- Different topologies can be obtained by varying the _____ of __, and we can select s so that the diameter of the network is _________.



## 4.2.6 Loop Networks –
### Diameter Formulation

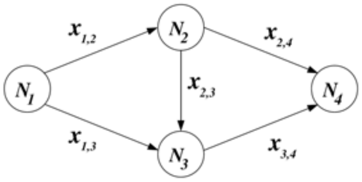
- Need D, diameter, in terms of s, _____ __________.
- Use a ________ ________ that uses ____________ chords
- If b is the number of backward chords used, and b’ the maximum number, then $b' = \left\lfloor \frac{n}{s+1} \right\rfloor$
- To these b’ links, we may need to add a maximum of s – 1 forward links, and thus, $D = \left\lfloor \frac{n}{s+1} \right\rfloor + (s-1)$
- The value $s = \lfloor \sqrt{n} \rfloor$ is optimal for most values of n yielding $Dopt \approx 2\sqrt{n} - 1$
- The value of s that ______________ the diameter also ____________ the number of __________ _____ and thus improves the ____________ of the network.

## 4.2.7 Ad Hoc Point-to-Point Networks

- The computing nodes in a _______________ computer system are quite often connected through a network that has no ____________ _____________.
- For this type of network, we would like to calculate the _____ __________, defined as the probability that there exists an _________________ ______ between two specific nodes, given the various _____ _________ probabilities.

## 4.2.7 Ad Hoc Point-to-Point Networks

### - Path Reliability, Example 1



For pair $N_1$-$N_4$

$P_1 = \{x_{1,2}, x_{2,4}\}$
$P_2 = \{x_{1,3}, x_{3,4}\}$
$P_3 = \{x_{1,2}, x_{2,3}, x_{3,4}\}$

- Let $p_{i,j}$ denote the probability that link $x_{i,j}$ is ________________ and $q_{i,j} = 1 - p_{i,j}$
- Any ______ ________ considered are incorporated into _______ _____.
- The disjoint events that allow N1 to send a message to N4 are (a) ________, (b) __________________, and (c) _______________________________

$$R_{N_1,N_4} = p_{1,2}p_{2,4} + p_{1,3}p_{3,4}[1 - p_{1,2}p_{2,4}] + p_{1,2}p_{2,3}p_{3,4}[q_{1,3}p_{2,4}]$$

## 4.2.7 Ad Hoc Point-to-Point Networks

### - Probability Inclusion and Exclusion

- Suppose for a given ___________________ _____, say $N_s$ and $N_d$, m paths, $P_1, P_2, \ldots, P_m$ exist from the _______ to the ______________.
- Denote by $E_i$ the _____ in which path $P_i$ is ___________.

$$R_{N_s,N_d} = \text{Prob}\{E_1 \cup E_2 \cup \ldots \cup E_m\}$$

- To get _________ sets,

$$E_1 \cup E_2 \cup \cdots \cup E_m = E_1 \cup (E_2 \cap \overline{E_1}) \cup (E_3 \cap \overline{E_1} \cap \overline{E_2}) \cup \cdots \cup (E_m \cap \overline{E_1} \cap \overline{E_2} \cap \cdots \cap \overline{E_{m-1}})$$

$$R_{N_s,N_d} = \text{Prob}\{E_1\} + \text{Prob}\{E_2 \cap \overline{E_1}\} + \cdots + \text{Prob}\{E_m \cap \overline{E_1} \cap \overline{E_2} \cap \cdots \cap \overline{E_{m-1}}\}$$

$$R_{N_s,N_d} = \text{Prob}\{E_1\} + \text{Prob}\{E_2\}\text{Prob}\{\overline{E_1} \mid E_2\} \cdot + \cdots + \text{Prob}\{E_m\}\text{Prob}\{\overline{E_1} \cap \overline{E_2} \cap \cdots \cap \overline{E_{m-1}} \mid E_m\}$$

## 4.2.7 Ad Hoc Point-to-Point Networks

### - Probability Inclusion and Exclusion

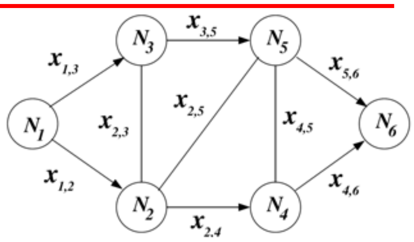
$$\mathrm{Prob}\{\overline{E_1} \cap \cdots \cap \overline{E_{i-1}} \mid E_i\} \qquad \mathrm{Prob}\{\overline{E_{1|i}} \cap \cdots \cap \overline{E_{i-1|i}}\}$$

- $\overline{E_{j|i}}$ is the ______ in which $P_j$ is ______ given that $P_i$ is ______________
- Define the ___________ ______ $P_{j|I} = P_j - P_i = \{x_k | x_k \in P_j, x_k \notin P_i\}$
- At least ____ ______ in $P_{j|I}$ must fail

## 4.2.7 Ad Hoc Point-to-Point Networks

### - Path Reliability Example 2

- Calculate the path reliability for the pair $N_1$-$N_6$
- Paths



$P_1 = \{x_{1,3}, x_{3,5}, x_{5,6}\}$
$P_2 = \{x_{1,2}, x_{2,5}, x_{5,6}\}$
$P_3 = \{x_{1,2}, x_{2,4}, x_{4,6}\}$
$P_4 = \{x_{1,3}, x_{3,5}, x_{4,5}, x_{4,6}\}$
$P_5 = \{x_{1,3}, x_{2,3}, x_{2,4}, x_{4,6}\}$
$P_6 = \{x_{1,3}, x_{2,3}, x_{2,5}, x_{5,6}\}$
$P_7 = \{x_{1,2}, x_{2,5}, x_{4,5}, x_{4,6}\}$
$P_8 = \{x_{1,2}, x_{2,3}, x_{3,5}, x_{5,6}\}$
$P_9 = \{x_{1,2}, x_{2,4}, x_{4,5}, x_{5,6}\}$
$P_{10} = \{x_{1,3}, x_{2,3}, x_{2,4}, x_{4,5}, x_{5,6}\}$
$P_{11} = \{x_{1,3}, x_{2,3}, x_{2,5}, x_{4,5}, x_{4,6}\}$
$P_{12} = \{x_{1,3}, x_{3,5}, x_{2,5}, x_{2,4}, x_{4,6}\}$
$P_{13} = \{x_{1,2}, x_{2,3}, x_{3,5}, x_{4,5}, x_{4,6}\}$

## 4.2.7 Ad Hoc Point-to-Point Networks - Path Reliability Example 2

- The conditional set $P_{1|2}$ is $P_{1|2} = \{x_{1,3}, x_{3,5}\}$ This set must fail in order for $P_1$ to be faulty while $P_2$ is working.
- $\mathrm{Prob}\{E1\} = p_{1,3}p_{3,5}p_{5,6}$
- $\mathrm{Prob}\{E_2\}\mathrm{Prob}\{(E_1)'|E_2\} = p_{1,2}p_{2,5}p_{5,6}(1 - p_{1,3}p_{3,5})$
- $\mathrm{Prob}\{E_3\}\mathrm{Prob}\{(E_1)'|E_3 \cap (E_2)'|E_3\}$
- $\mathrm{Prob}\{E_4\}\mathrm{Prob}\{(E_1)'|E_4 \cap (E_2)'|E_4 \cap (E_3)'|E_4\}$

---

## 4.3 Fault Tolerant Routing – Introduction

- The ____________ of a fault-tolerant routing strategy is to get a message from source to destination despite a _______ ___ ___ _________ being faulty.
- The focus here is on ______ routing, ____ _____, as opposed to ________, __________ _______.
- Routing algorithms can either be ____________ or ________________
- The route can be chosen __________ or ____________.
- Consider routing approaches for the ____________ ____________ and the ____________ ______.

## 4.3.1 Hypercube Fault-Tolerant Routing

- The routing algorithm must be ________ to continue to successfully route messages in __________hypercubes.
- Let TD denote the _______ ____ ___________ that the message has already traversed.
- $TD^R$ is the _____ __________.
-        denotes the _____ __________ carried out k times, sequentially
- If D is the ____________ and S the ________, let $d = D \oplus S$, where is a _________ ____ _____________ on D and S, relative address of D with respect to S
- Let SR(TD) be the set of __________ _______________ reachable by traversing each of _____ _____________ _______ in TD, in that order.
- Denote $e_n^i$ the n-bit vector consisting of a 1 in the ______ _________ and _____ everywhere else
- Messages consist of __, __________, _____
- TRANSMIT(j) – Send the message $(d \oplus e^j$, payload, $TD \odot j)$ along the j-th dimension link, $\odot$ means append

## 4.3.1 Hypercube Fault-Tolerant Routing

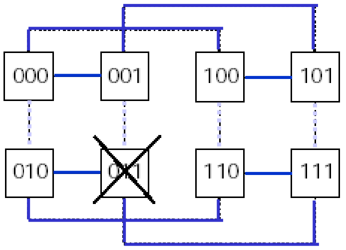
```
If (d == 0•••0)
   Accept message and Exit algorithm
else
{
  for j = 0 to (n – 1) step 1 do
  {
    if ((d_j == 1) && (jth link good) && (e_n^j ∉ SR(TDR))
    {
      TRANSMIT(j)
      Exit algorithm
    }
  }
}
```

## 4.3.1 Hypercube Fault-Tolerant Routing

```
//If we are not done at this poitn, it means that there is no
//way of getting one step closer to the destination from this
//node; we need to take a detour.
if (non-faulty link not in SR(TD^R)
  Pick one link h
else
  Define g = max
  if (g == number of elements in SR(TD))
    Give up
    Exit algorithm
  else
    h = element (g +1) in TDR
  end if
  TRANSMIT(h)
end if
```

## 4.3.1 Hypercube Fault-Tolerant Routing - Example

- H3 with faulty node 011
- S = 000 wants to send a message to D = 111



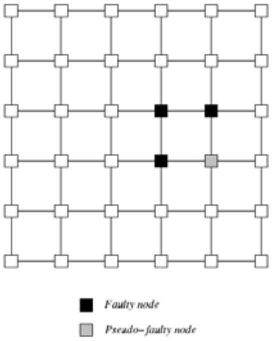
## 4.3.2 Origin-Based Routing in the Mesh

- The algorithm used for the hypercube uses a ______ _____ searching strategy and ______________as appropriate.
- If the faulty regions are known ___ ___________, no _____________ will occur, this approach is taken here.
- Consider a two-dimensional rectangular _____ mesh with at most _____ failures.
- This procedure can be extended to _____ ____________ and ______ than _____ failures.
- Assumption: All faulty regions are _______, some good nodes may be _________ ________ to make the region ________, these are ______________ nodes.
- Each node knows the ___________ along each direction (east, west, north, and south) to the nearest __________ ________ in that direction.

---

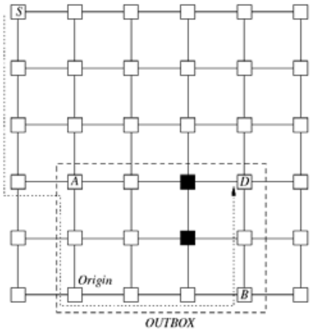
## 4.3.2 Origin-Based Routing in the Mesh

- Define ____ _______ as the origin.
- With no more than ____ failures, the origin node can be chosen so that its _____ and _______ do not have any _________ _______.
- The path from node S to node D is divided into an ________, edges that get ________ to the origin and an _________, edges that get ____________ _______ from the origin, either of the sets can be ________.



## 4.3.2 Origin-Based Routing in the Mesh

- The destination node D has an _______ associated with it that is the smallest rectangular region that contains within it both the ______ and the ____________.
- A node V is ____ with respect to ___________ ___ and some set of ________ ______, ___, if
  - Node V is in the ______ ____ __ and
  - __, __ and an ________ from __ to __ are fault-free



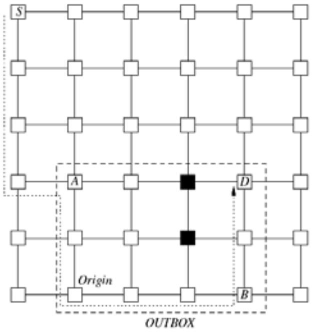
- Denote by ($x_A$, $y_A$) the Cartesian coordinates of node A, then the __________ ____ for a destination node D is the set of all nodes in the ________ ____ __ satisfying the condition that

$$x_V - y_D = X_D - y_D + e \text{ where } e \in \{-1, 0, 1\}$$

---

## 4.3.2 Origin-Based Routing in the Mesh - Routing Algorithm

- Phase 1. The message is routed on an _______ until it reaches the _______ and message is in node U.
- Phase 2. Compute the ________ from U to the ________ _____ node in each direction, compute the distance to the nearest _____ _____ in each direction. If ____ _____ closer than ________, route to the _____ _____, otherwise continue to route on the ___ _____.



- Phase 3. If the safe node U has a safe, __________ neighbor V closer to the destination, send it to V, otherwise _____ faulty region and turn ________ ____ ___________ _____ when it arrives at the ______ of the faulty square.