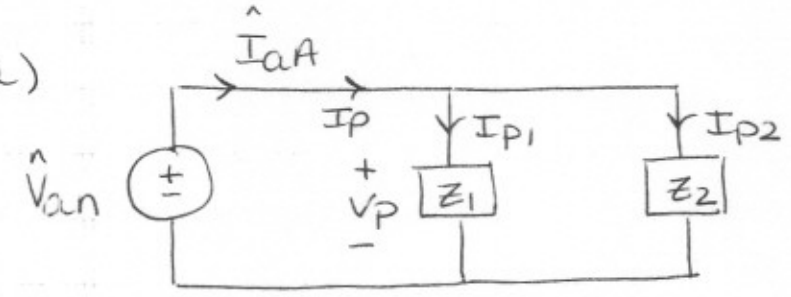
Problem Set #5 - Solutions pg 1

① a)



$V_L = 400$ Vrms

Z_1: $P_{3\phi_1} = 15$ kW $PF_1 = 0.25$ lag

Z_2: $\hat{S}_{3\phi_2} = 60\angle 50°$ kVA

$$\left.\begin{aligned}\hat{V}_{ab} &= 400\angle 30° \\ \hat{V}_{bc} &= 400\angle -90° \\ \hat{V}_{ca} &= 400\angle +150°\end{aligned}\right\} \text{Vrms}$$

$\hat{V}_{an} = 230.94\angle 0°$ Vrms

note:

b) $V_p = |\hat{V}_{an}| = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}}$

$V_p = 230.94$ Vrms

c) $\angle Z_1 = \cos^{-1}(.25) = 75.52°$

$\angle Z_2 = 50°$

$P_{3\phi_1} = \sqrt{3}\, V_L\, I_{L1} \cdot PF_1$

$$I_{L1} = \frac{15 \times 10^3}{\sqrt{3} \cdot 400 \cdot 0.25} = 86.60 \text{ Arms}$$

$\hat{I}_{L1} = 86.60\angle -75.52°$ Arms

$\hat{S}_{3\phi_2} = \sqrt{3}\, V_L\, I_{L2} \angle Z_L$

$\sqrt{3}\, V_L\, I_{L2} = 60 \times 10^3$

$$I_{L2} = \frac{60 \times 10^3}{\sqrt{3}(400)} = 86.60 \text{ Arms}$$

$\hat{I}_{L2} = 86.60\angle -50°$ Arms

$\hat{I}_{aA} = \hat{I}_{L1} + \hat{I}_{L2} = 168.92\angle -62.76°$ Arms

$\hat{I}_{bB} = 168.92\angle 177.24°$ Arms

$\hat{I}_{cC} = 168.92\angle 57.24°$ Arms

① cont d) $I_p = |\hat{I}_{aA}| = I_L = 168.92 \text{ Arms}$

e) $Z_1 = \dfrac{\hat{V}_{an}}{\hat{I}_{L1}} = \dfrac{230.94\angle 0^\circ}{86.60\angle -75.52^\circ} = 2.67\angle 75.52^\circ \Omega$

$Z_2 = \dfrac{\hat{V}_{an}}{\hat{I}_{L2}} = \dfrac{230.94\angle 0^\circ}{86.60\angle -50^\circ} = 2.67\angle 50^\circ \Omega$

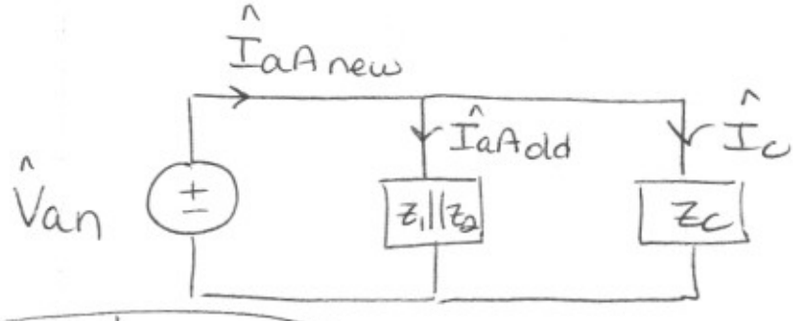
f) $P_{3\phi_S} = \sqrt{3} V_L I_L \cdot PF_{1-2} = \sqrt{3} V_L I_L \cos(\theta - \phi)$

$P_{3\phi_S} = \sqrt{3}(400)(168.92)\cos(62.76)$
$= 53.57 \text{ kW}$

check $P_{3\phi_S} = P_{3\phi_1} + P_{3\phi_2}$ $\quad P_{3\phi_2} = Re[\hat{S}_{3\phi_2}]$
$= 38.57 \text{ kW}$

$= 15 + 38.57$
$= 53.57 \text{ kW}$ ☺

g)



$PF_{CL} = 0.95 \text{ lag}$
$\angle Z_{CL} = \cos^{-1}(.95)$
$= 18.19^\circ$

$I_{Lnew} = |\hat{I}_{aA\,new}|$

$P_{3\phi_S} = \sqrt{3} V_L I_{Lnew} \cdot PF_{CL}$

$I_{Lnew} = \dfrac{53.57 \times 10^3}{\sqrt{3}(400)(.95)} = 81.39 \text{ Arms}$

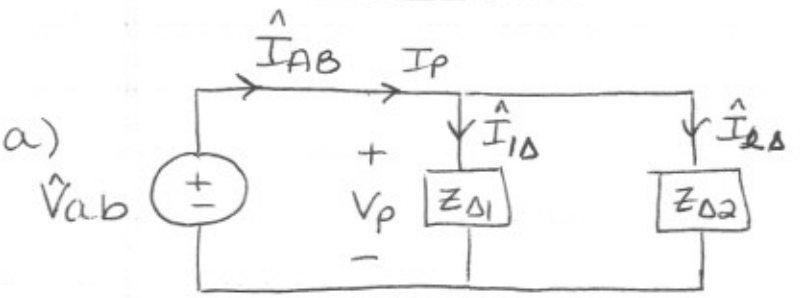
① cont

$$\hat{I}_{aA\,new} = 81.39\angle -18.19^\circ \text{ Arms}$$

$$\hat{I}_C = \hat{I}_{aA\,new} - \hat{I}_{aA\,old} = 124.78\angle 90^\circ \text{ Arms}$$

$$Z_C = \frac{\hat{V}an}{\hat{I}_C} = 1.85\angle -90^\circ \Omega$$

②



a)

$|\hat{V}an| = 100$ Vrms

$Z_{\Delta 1}$: $\hat{S}_{3\Phi_1} = 3.3\angle 75^\circ$ kVA

$Z_{\Delta 2}$: $\hat{S}_{3\Phi_2} = 6\angle 10^\circ$ kVA

a) $V_L = \sqrt{3}\,|\hat{V}an| = 173.20$ Vrms

$\hat{V}ab = 173.20\angle 30^\circ$ Vrms

$\hat{V}bc = 173.20\angle -90^\circ$ Vrms

$\hat{V}ca = 173.20\angle +150^\circ$ Vrms

$\hat{V}an = 100\angle 0^\circ$ Vrms

b) $V_P = V_L = 173.20$ Vrms

c) $\hat{I}_{aA} = \hat{I}_{AB} - \hat{I}_{CA}$

$\hat{I}_{AB} = \hat{I}_{1\Delta} + \hat{I}_{2\Delta}$

$$\hat{S}_{1\Phi_1} = \frac{\hat{S}_{3\Phi_1}}{3} = 1.1\angle 75^\circ \text{ kVA}$$

$$\hat{S}_{1\Phi_1} = \hat{V}ab \cdot \hat{I}_{1\Delta}^{*} = 1.1\angle 75^\circ \text{ kVA}$$

$$\hat{I}_{1\Delta} = 6.35\angle -45^\circ \text{ Arms}$$

(2) cont c) $\hat{S}_{1\phi_2} = 2\angle 10^\circ$ kVA

$\hat{V}_{ab} \cdot \hat{I}_{2\Delta}^* = 2\angle 10^\circ$ kVA

$\hat{I}_{2\Delta} = 11.55\angle 20^\circ$ Arms

$\hat{I}_{AB} = \hat{I}_{1\Delta} + \hat{I}_{2\Delta} = 15.35\angle -2.02$ Arms

$\hat{I}_{CA} = 15.35\angle 117.98^\circ$

$\hat{I}_{aA} = 26.59\angle -32.02^\circ$ Arms

$\hat{I}_{bB} = 26.59\angle -152.02^\circ$ Arms

$\hat{I}_{cC} = 26.59\angle 87.98^\circ$ Arms

$I_L = \sqrt{3} I_p$

$= \sqrt{3}(15.35)$

$= 26.59$ Arms

d) $I_p = |\hat{I}_{AB}| = \frac{I_L}{\sqrt{3}} = 15.35$ Arms

e) $Z_{1\Delta} = \frac{\hat{V}_{ab}}{\hat{I}_{1\Delta}} = 27.27\angle 75^\circ\ \Omega$

$Z_{2\Delta} = \frac{\hat{V}_{ab}}{\hat{I}_{2\Delta}} = 15\angle 10^\circ\ \Omega$

f) $P_{3\phi_s} = \sqrt{3} V_L I_L \cos(\theta - \phi)$

$= \sqrt{3}(173.20)(26.59)\cos(32.02^\circ)$

$= 6.76$ kW

check $P_{3\phi_1} = \mathrm{Re}[\hat{S}_{3\phi_1}] = 0.854$ kW

$P_{3\phi_2} = \mathrm{Re}[\hat{S}_{3\phi_2}] = 5.91$ kW

$\Big\} 6.76$ kW ☺

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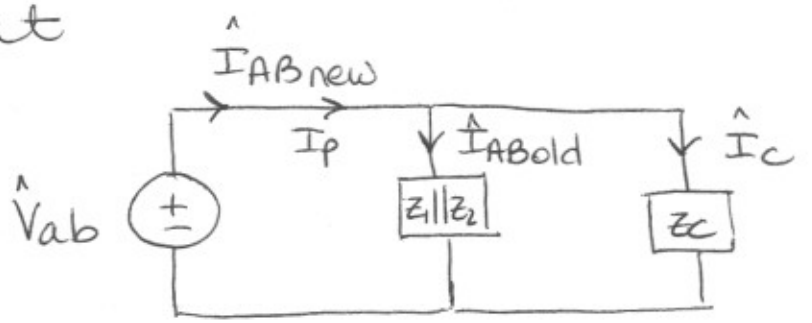
(2) cont

g)



$PF_{CL} = 0.8$ lag

$\angle Z_{CL} = 36.87°$

$P_{3\phi s} = 6.76$ kW

$$6.76 \times 10^3 = \sqrt{3}\, V_L\, I_{Lnew} \cdot PF_{aL}$$

$$I_{Lnew} = \frac{6.76 \times 10^3}{\sqrt{3}(173.20)(.8)} = 28.17 \text{ Arms}$$

$$I_{Pnew} = \frac{I_{Lnew}}{\sqrt{3}} = 16.26 \text{ Arms}$$

$$\hat{I}_{AB\,new} = I_{Pnew}\left(\angle(30° - \angle Z_{CL})\right)$$

$$= 16.26 \angle -6.87° \text{ Arms}$$

$$\hat{I}_C = \hat{I}_{AB\,new} - \hat{I}_{AB\,old}$$

$$= 1.62 \angle -60° \text{ Arms}$$

$$Z_C = \frac{\hat{V}_{ab}}{\hat{I}_C} = \frac{173.20 \angle 30}{1.62 \angle -60°} = 107.10 \angle 90° \,\Omega$$

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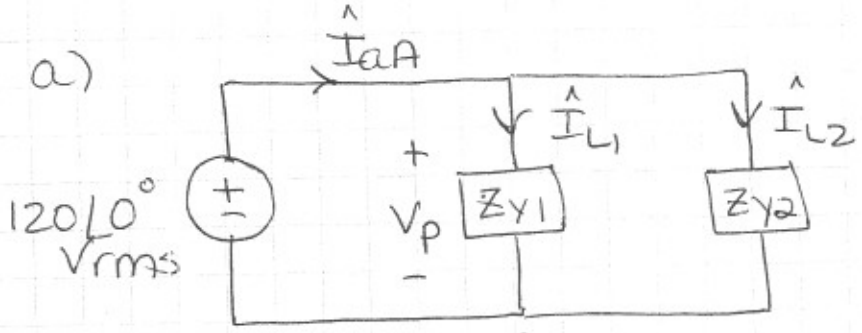
③ $|\hat{V}_{an}| = 120$ Vrms

$Z_\Delta \Rightarrow P_\Delta = 24$ kW $\quad PF_\Delta = 0.468$ lag

$Z_Y \Rightarrow \hat{S}_Y = 12 \angle 30^\circ$ kVA

a)



↑ assume Y equivalent of Δ-load

b) $\hat{V}_{ab} = \sqrt{3}(120)\angle 30^\circ = 207.85 \angle 30^\circ$ Vrms

$\hat{V}_{bc} = 207.85 \angle -90^\circ$ Vrms

$\hat{V}_{ca} = 207.85 \angle +150^\circ$ Vrms

c) $V_p = |\hat{V}_{an}| = 120$ Vrms

d) find $\hat{I}_{L1}$ & $\hat{I}_{L2}$, first.

$P_{Y1} = 24$ kW $\quad PF_{Y1} = 0.468$ lag $\Rightarrow \angle Z_{Y1} = 62.10^\circ$

$24 \times 10^3 = \sqrt{3}\, V_L I_{L1} \cdot PF_1$

$I_L = 142.45$ Arms

$\hat{I}_{L1} = 142.45 \angle -62.10^\circ$ Arms

$\hat{S}_{Y2} = 12 \times 10^3 \angle 30^\circ \quad \angle Z_{Y2} = 30^\circ$

$12 \times 10^3 = \sqrt{3} V_L I_{L2} \quad I_{L2} = 33.33$ Arms

(3) $\hat{I}_{L2} = 33.33 \angle -30^\circ$ Arms

$\hat{I}_{aA} = \hat{I}_{L1} + \hat{I}_{L2} = 171.60 \angle -56.18^\circ$ Arms

$\hat{I}_{bB} = 171.60 \angle -176.18^\circ$ Arms

$\hat{I}_{cC} = 171.60 \angle 63.82^\circ$ Arms

-or- $P_{Y1} = 24$ kW $PF_1 = 0.468$ $\angle Z_{Y1} = 62.10$

$\angle Z_{Y1} = \angle \hat{S}_{Y1} = 62.10$

→ $|\hat{S}_{Y1}| = \sqrt{3} V_L I_{L1} = \frac{P_{Y1}}{PF_1} = \frac{24 \times 10^3}{0.468} = 51.28$ kVA

so $\hat{S}_{Y1} = 51.28 \angle 62.10^\circ$ kVA

$\hat{S}_{total} = \hat{S}_{Y1} + \hat{S}_{Y2} = 51.28 \angle 62.10^\circ + 12 \angle 30^\circ$

$= 61.77 \angle 56.18^\circ$ kVA

$|\hat{S}_{total}| = \sqrt{3} V_L I_L$, where $I_L = |\hat{I}_{aA}|$

$I_L = 171.59$ Arms (angle is opposite sign of the angle of $\hat{S}_{total}$)

$\hat{I}_{aA} = 171.59 \angle -56.18^\circ$ Arms ✓ checks!

e) $I_P = I_L$ for Y-Y systems, where $I_L = |\hat{I}_{aA}|$

$= 171.60$ Arms

f) $Z_{Y1} = \frac{\hat{V}_{an}}{\hat{I}_{L1}} = 0.84 \angle 62.10^\circ \Omega$

$Z_{Y2} = \frac{\hat{V}_{an}}{\hat{I}_{L2}} = 3.6 \angle 30^\circ \Omega$

Note:
you can calculate the Δ-equivalent of these loads

$Z_{\Delta 1} = 3Z_{Y1} = 2.53 \angle 62.10^\circ \Omega$

$Z_{\Delta 2} = 3Z_{Y2} = 10.80 \angle 30^\circ \Omega$

g) $P_{3\phi s} = \sqrt{3}\, V_L I_L \cdot \cos(\theta - \phi)$

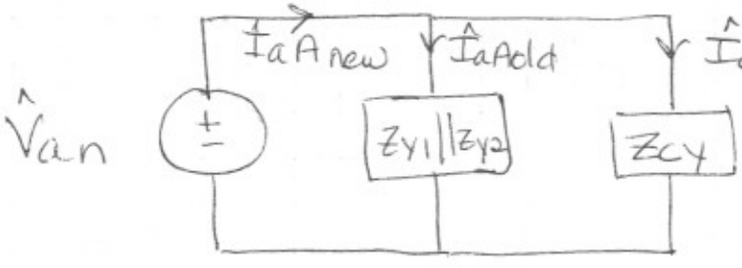
$= \sqrt{3}(207.85)(171.60)\cos(0 - (-56.18))$

$= 34.38$ kW

or from $\hat{S}_{total} = 61.77 \angle 56.18^\circ$ kVA

$P_{3\phi s} = Re[\hat{S}_{total}] = 34.38$ kW checks ☺

h) Again assume a y-equivalent load.



$\hat{I}_{aA\,old} = 171.60 \angle -56.18^\circ$ Arms

$P_{3\phi} = 34.38$ kW

$PF_{CL} = 0.9$ lag

$\angle Z_{CL} = \cos^{-1}(.9) = 25.84^\circ$

$34.38 \times 10^3 = \sqrt{3}\, V_L I_{L\,new} \cdot PF_{CL}$

$I_{L\,new} = 106.12$ Arms

$\hat{I}_{aA\,new} = 106.12 \angle -25.84$

$\hat{I}_c = \hat{I}_{aA\,new} - \hat{I}_{aA\,old}$

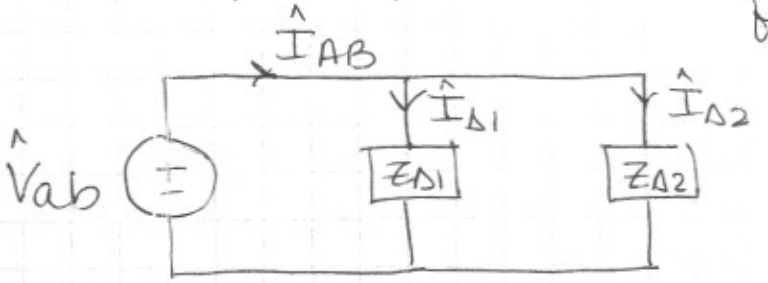
$= 96.31 \angle 90^\circ$ Arms

$Z_{CY} = \hat{V}_{an} - 1.25 \angle -90^\circ \Omega$

h) cont $\underline{Z_{CY} = 1.25\angle -90^\circ\ \Omega}$

$$\boxed{Z_{C\Delta} = 3Z_{CY} = 3.74\angle -90^\circ\ \Omega}$$

repeat for Y-Δ. (I will refer to some answers from previous Y-Y work)



b) $\hat{V}_{ab} = 207.85\angle 30^\circ$ Vrms

c) for Y-Δ $V_p = V_L = \sqrt{3}\,|\hat{V}_{an}| = |\hat{V}_{ab}| = 207.85$ Vrms

d) $\hat{I}_{aA} = \hat{I}_{AB} - \hat{I}_{CA}$

find $\hat{I}_{AB}$ by find $\hat{I}_{\Delta 1}$ and $\hat{I}_{\Delta 2}$

$Z_{\Delta 1}$: $P_{3\phi_1} = 24$ kW $PF_1 = 0.468$ $\angle Z_{\Delta 1} = 62.10$

$24\times10^3 = \sqrt{3}\,V_L I_{L1} \cdot PF_1$ $I_{L1} = 142.45$ Arms

$I_{p1} = |\hat{I}_{\Delta 1}| = \frac{I_{L1}}{\sqrt{3}} = 82.24$ Arms

$\angle \hat{I}_{\Delta 1} = 30 - \angle Z_{\Delta 1}$
$= 30 - 62.10 = -32.10$

$$\boxed{\hat{I}_{\Delta 1} = 82.24\angle -32.10^\circ \text{ Arms}}$$

$\hat{I}_{\Delta 2} \Rightarrow I_{L2} = 33.33$ Arms $\angle Z_{\Delta 1} = 30^\circ$ (from Y-Y)

$I_{p2} = \frac{I_{L2}}{\sqrt{3}} = 19.24$ Arms $\angle I_{\Delta 2} = 30 - 30^\circ = 0^\circ$

$$\boxed{\hat{I}_{\Delta 2} = 19.24\angle 0^\circ \text{ Arms}}$$

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③ cont

d) $\hat{I}_{AB} = \hat{I}_{\Delta 1} + \hat{I}_{\Delta 2} = 99.07 \angle -26.17^\circ$ Arms

$\hat{I}_{BC} = 99.07 \angle 146.17^\circ$ Arms

$\hat{I}_{CA} = 99.07 \angle -93.83^\circ$ Arms

$\hat{I}_{aA} = \hat{I}_{AB} - \hat{I}_{CA} = 171.59 \angle -56.17^\circ$ Arms

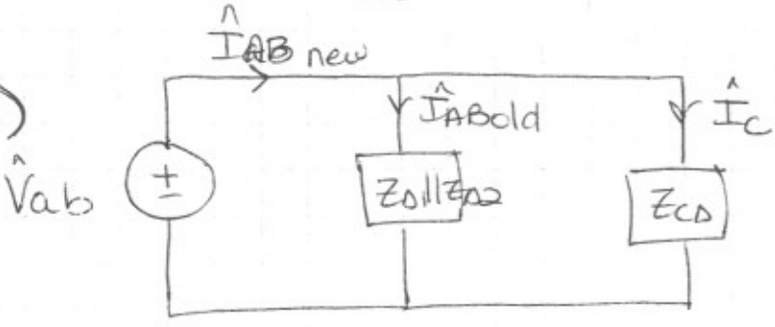
(looks familiar ☺)

e) $I_p = |\hat{I}_{AB}| = \frac{I_L}{\sqrt{3}} = 99.07$ Arms

f) $Z_{\Delta 1} = \frac{\hat{V}ab}{\hat{I}_{\Delta 1}} = 2.53 \angle 62.10^\circ \Omega$

$Z_{\Delta 2} = \frac{\hat{V}ab}{\hat{I}_{\Delta 2}} = 10.80 \angle 30^\circ \Omega$

(also familiar ☺)



$\hat{I}_{ABold} = 99.07 \angle -26.17^\circ$ Arms

$P_{3\phi}$ (from before) $= 34.38$ kW

$PF_{CL} = 0.9$ lag

$34.38 \times 10^3 = \sqrt{3} V_L I_{L new} \cdot PF_{CL}$

$\angle Z_{CL} = 25.84^\circ$

$I_{L new} = 106.12$ Arms

$I_{p new} = |\hat{I}_{AB new}| = \frac{106.12}{\sqrt{3}} = 61.27$ Arms

$\angle \hat{I}_{AB new} = 30 - \angle Z_{CL}$
$= 30 - 25.84$
$= 4.16^\circ$

$\hat{I}_{AB new} = 61.27 \angle 4.16^\circ$ Arms

$\hat{I}_C = \hat{I}_{AB new} - \hat{I}_{ABold}$

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(3) $\hat{I}_C = 55.59 \angle 120^\circ$ Arms

$Z_{C\Delta} = \frac{\hat{V}ab}{\hat{I}_C} = 3.74 \angle -90^\circ \ \Omega$

note $Z_{C\Delta} = 3Z_{cy}$ from previous work

(4) $I_L = 300$ Arms

Z_1: 12 kW PF = 0.257 lag

Z_2: 6 kW 15 kVAR

~~$\hat{S}_1 \Rightarrow \frac{12\text{kW}}{.257} =$~~

$|\hat{S}_1| = \frac{P_1}{PF_1} = \frac{12 \times 10^3}{.257} = 46.69$ kVA

$L\hat{S}_1 = \cos^{-1}(.257) = 75.10^\circ$

$\hat{S}_1 = 46.69 \angle 75.10^\circ$ kVA

$\hat{S}_2 = P_2 + jQ_2 = 16.15 \angle 68.20^\circ$ kVA

$\hat{S}_{3\phi S} = \hat{S}_{13\phi} + \hat{S}_{23\phi} = 62.76 \angle 73.33^\circ$ kVA

$|\hat{S}_{3\phi S}| = 62.76 \times 10^3 = \sqrt{3} V_L I_L$

$V_L = 120.78$ Vrms

$\hat{V}ab = 120.78 \angle 30^\circ$ Vrms

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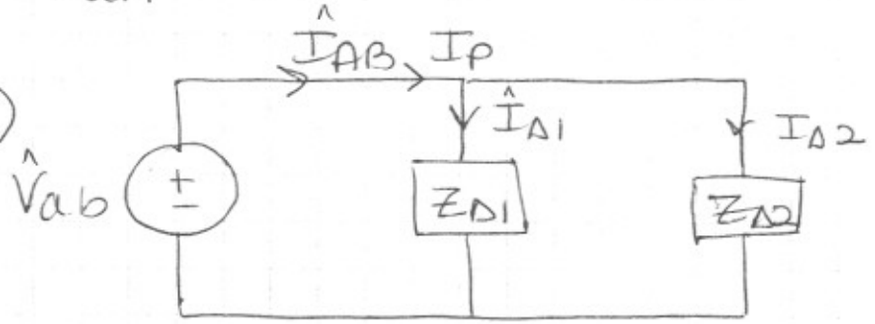
④ cont

$$|\hat{V}an| = \frac{V_L}{\sqrt{3}} = 69.73 \text{ Vrms}$$

$$\hat{V}an = 69.73 \angle 0^\circ \text{ Vrms}$$

b)



$\hat{I}_{\Delta 1}$: $12 \times 10^3 = \sqrt{3}\, V_L I_{L1} \cdot PF_1$ $\quad \angle Z_1 = 75.10^\circ$

$I_{L1} = 223.20$ Arms $\quad \angle I_{\Delta 1} = 30 - 75.10^\circ = -45.10^\circ$

$$I_{P1} = \frac{I_{L1}}{\sqrt{3}} = 128.86 \text{ Arms}$$

$$\hat{I}_{\Delta 1} = 128.86 \angle -45.10^\circ \text{ Arms}$$

$\hat{I}_{\Delta 2}$: $\hat{S}_2 = 16.15 \angle 68.20^\circ$ KVA

$16.15 \times 10^3 = \sqrt{3} V_L I_{L2}$ $\quad I_{L2} = 77.20$ Arms

$$I_{P2} = \frac{I_{L1}}{\sqrt{3}} = 44.57 \text{ Arms} \quad \angle I_{\Delta 2} = 30 - 68.20 = -38.20$$

$$\hat{I}_{\Delta 2} = 44.57 \angle -38.20^\circ \text{ Arms}$$

$$\hat{I}_{AB} = 44.57 \angle -38.20 + 128.86 \angle -45.10^\circ$$

$$= 173.19 \angle -43.33^\circ \text{ Arms}$$

$$\hat{I}_{aA} = 223.20 \angle -75.10^\circ + 77.20 \angle -68.20^\circ$$

$$= 299.98 \angle -73.33^\circ \text{ Arms}$$

↑ close enough to 300 Arms which was given

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(4) cont

b) $I_p = |\hat{I}_{AB}| = 173.19$ Arms

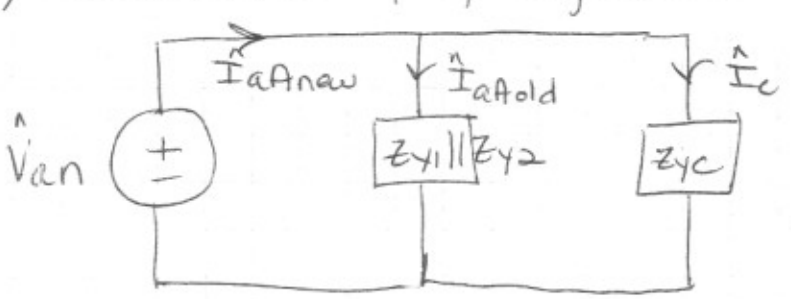
c) $Z_{\Delta 1} = \frac{\hat{V}_{ab}}{\hat{I}_{\Delta 1}} = 0.9\text{H} \angle 75.10^\circ\ \Omega$

$Z_{\Delta 2} = \frac{\hat{V}_{ab}}{\hat{I}_{\Delta 2}} = 2.71 \angle 68.20^\circ\ \Omega$

d) $Z_{Y1} = \frac{\hat{V}_{an}}{\hat{I}_{L1}} = \frac{1}{3} Z_{\Delta 1} = 0.31 \angle 75.10^\circ\ \Omega$

$Z_{Y2} = \frac{\hat{V}_{an}}{\hat{I}_{L2}} = \frac{1}{3} Z_{\Delta 2} = 0.90 \angle 68.20^\circ\ \Omega$

e) assume Y-Y system



$\hat{I}_{aAold} = 300 \angle -73.33^\circ$ Arms

$P_{S3\phi} = Re[\hat{S}_{3\phi s}]$
$= 18\ kW$
$= P_{3\phi 1} + P_{3\phi 2}$

$PF_{CL} = 1$
$Z_{CL} = 0^\circ$

$18 \times 10^3 = \sqrt{3}\, V_L I_{Lnew} \cdot 1$

$I_{Lnew} = 86.04$ Arms

$\hat{I}_{aAnew} = 86.04 \angle 0^\circ$ Arms

$\hat{I}_c = \hat{I}_{aAnew} - \hat{I}_{aAold} = 287.35 \angle 90^\circ$ Arms

$$Z_{cy} = \frac{\hat{V}_{an}}{\hat{I}_c} = 0.243 \angle -90^\circ\ \Omega$$