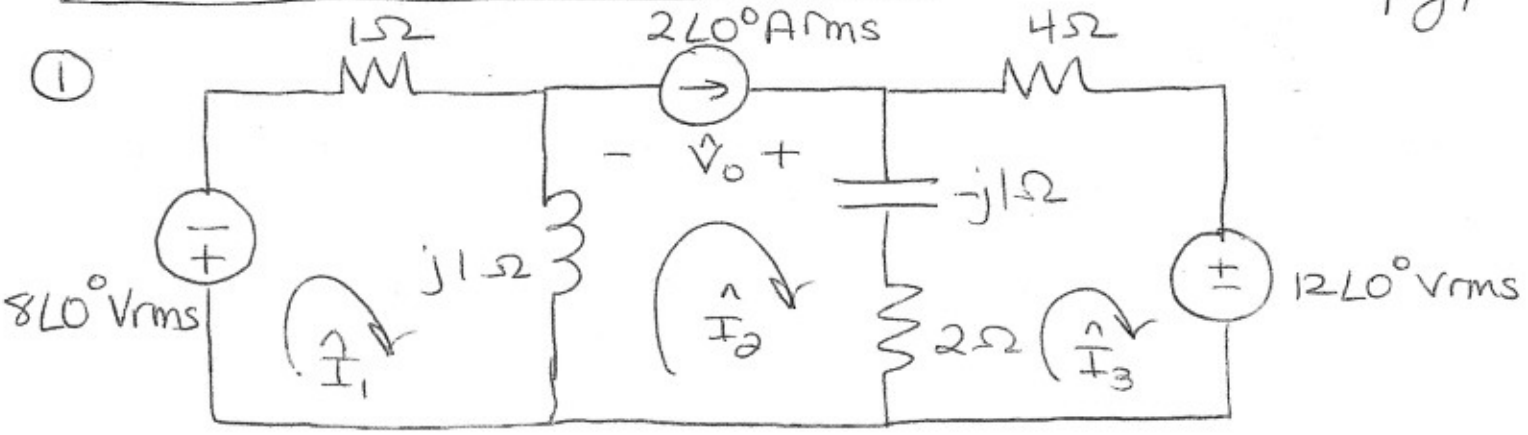
Problem Set 3 - Solutions

①



Know: $\hat{I}_2 = 2\angle 0^\circ$ Arms

m1: $-8\angle 0^\circ - 1\hat{I}_1 - j1(\hat{I}_1 - \hat{I}_2) = 0$

m2: $-(j1)(\hat{I}_2 - \hat{I}_1) + \hat{V}_o - (2 - j1)(\hat{I}_2 - \hat{I}_3) = 0$

m3: $-(2 - j1)(\hat{I}_3 - \hat{I}_2) - 4\hat{I}_3 - 12\angle 0^\circ = 0$

Simplify: (m1) $(-1 - j1)(\hat{I}_1) = 8\angle 0^\circ - j1(\hat{I}_2)$

$(-1 - j1)\,\hat{I}_1 = 8.25\angle -14.04$

$$\hat{I}_1 = 5.83\angle 120.96^\circ \text{ Arms}$$

(m3) $\hat{I}_3(-6 + j1) = 12\angle 0^\circ - (2 - j1)(\hat{I}_2)$

$$\hat{I}_3 = 1.36\angle -156.50^\circ \text{ Arms}$$

(m2) $\hat{V}_o = j1(\hat{I}_2 - \hat{I}_1) + (2 - j1)(\hat{I}_2 - \hat{I}_3)$

$$\hat{V}_o = 12.37\angle 13.26^\circ \text{ Vrms}$$

① cont

8L0° Vrms: $P = V_{rms} I_{rms} \cos(\theta - \phi)$ ($I_{rms} = |\hat{I}_1|$)

$= 8(5.83)\cos(0 - 120.96°)$

$P = -24$ W, Abs

$Q = V_{rms} I_{rms} \sin(\theta - \phi)$

$= 8(5.83)\sin(-120.96°)$

$Q = -40$ VAR, Abs

$\hat{S} = (8\angle 0)(\hat{I}_1)^*$

$= (8\angle 0)(5.83\angle -120.96°)$

$\hat{S} = 46.64\angle -120.96°$ VA, Abs

1Ω: $P = |\hat{I}_1|^2 \cdot 1 = (5.83)^2(1) = 34$ W, Abs

$Q = 0$ VAR

$\hat{S} = 34\angle 0°$ VA

j1Ω: $P = 0$ W

$Q = 1(|\hat{I}_1 - \hat{I}_2|)^2 = 1(7.07)^2 = 50$ VAR, Abs

$\hat{S} = 50\angle 90°$ VA abs

2L0° Arms: $P = V_0 \cdot 2 \cdot \cos(\theta - \phi)$

$= (12.37)(2)\cos(13.26°)$

$P = 24.08$ W, Del

$\hat{S} = (12.37\angle 13.26°)(2\angle 0)$

$= 24.74\angle 13.26°$ VA Del

$Q = (12.37)(2)\sin(13.26°)$

$Q = 5.67$ VAR, Del

2Ω: $P = 2(|\hat{I}_2 - \hat{I}_3|)^2$

$= 2(3.29)^2$

$= 21.68$ W, Abs

$Q = 0$ VAR

$\hat{S} = 21.68\angle 0°$ VA, Abs

① cont

$-j1\Omega$: $P = 0$ W

$Q = -1(|\hat{I}_2 - \hat{I}_3|)^2$

$= -1(3.29)^2$

$= -10.82$ VAR, Abs

$\hat{S} = 10.82 \angle -90^\circ$ VA

$12\angle 0^\circ$ Vrms: $P = 12 I_3 \cos(0 + 156.50)$

$= 12(1.36)\cos(156.50)$

$P = -14.97$ W, Abs

$Q = 12 I_3 \sin(156.50)$

$Q = 6.51$ VAR, Abs

$\hat{S} = (12\angle 0)(1.36\angle 156.50)$

$= 16.32\angle 156.50^\circ$ VA, Abs

4Ω: $P = 4|\hat{I}_3|^2 = 4(1.36)^2 = 7.40$ W, Abs

$Q = 0$ VAR, Abs

$\hat{S} = 7.40\angle 0^\circ$ VA, Abs

$\Sigma P_{del} = 24.08$ W

$\Sigma Q_{del} = 5.67$ VAR

$\Sigma \hat{S}_{del} = 24.74\angle 13.26^\circ$ VA

checks ☺

$\Sigma P_{abs} = -24 + 34 + 21.68 - 14.97 + 7.40$

$\boxed{\Sigma P_{abs} = 24.11 \text{ W}}$

$\Sigma Q_{abs} = -40 + 50 - 10.82 + 6.51$

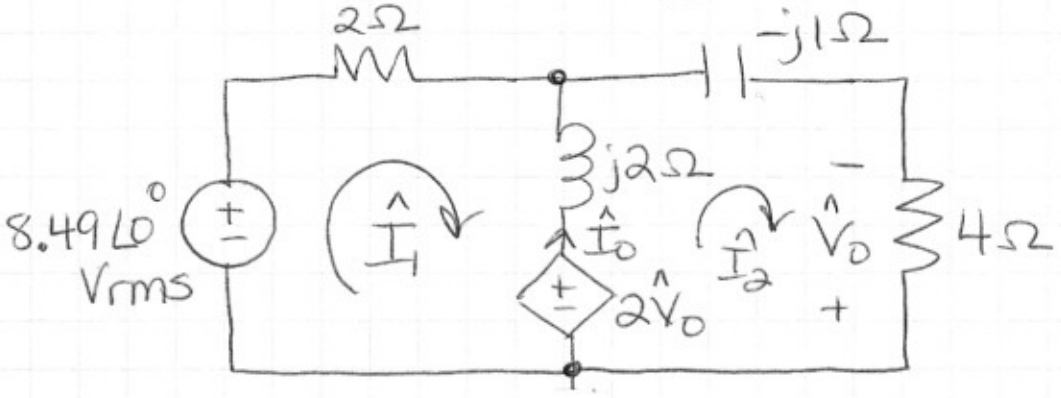
$\boxed{\Sigma Q_{abs} = 5.69 \text{ VAR}}$

$\Sigma \hat{S}_{abs} = 24.78\angle 13.28$ VA

$\Sigma \hat{S}_{abs} = (46.64\angle -120.96^\circ) + (34\angle 0) + (50\angle 90) + (21.68\angle 0)$

$+ 10.82\angle -90 + 16.32\angle 156.50 + 7.40\angle 0$

(2) $V_m = 12 \quad V_{rms} = \frac{12}{\sqrt{2}} = 8.49 \text{ Vrms}$



$\hat{V}_0 = -4\hat{I}_2$

(note: $\hat{I}_0$ labelled)

m1: $8.49\angle 0 - 2\hat{I}_1 - j2(\hat{I}_1 - \hat{I}_2) - 2\hat{V}_0 = 0$

m2: $2\hat{V}_0 - j2(\hat{I}_2 - \hat{I}_1) - (-j1)\hat{I}_2 - 4\hat{I}_2 = 0$

m1: $\hat{I}_1(-2-j2) + \hat{I}_2(8+j2) = 8.49\angle 180$

m2: $\hat{I}_1(j2) + \hat{I}_2(-12-j1) = 0$

$\hat{I}_1 = 3.67\angle -16.27^\circ \text{ Arms} \qquad \hat{V}_0 = 2.44\angle -111.04^\circ \text{ Vrms}$

$\hat{I}_2 = 0.61\angle 68.96^\circ \text{ Arms}$

power

2Ω: $P = |\hat{I}_1|^2 \cdot 2 = 26.94 \text{ W, Abs}$ | 4Ω: $P = |\hat{I}_2|^2 \cdot 4 = 1.49 \text{ W, Abs}$

$Q = 0$ | $Q = 0$

$\hat{S} = 26.94\angle 0^\circ \text{ VA, Abs}$ | $\hat{S} = 1.49\angle 0^\circ \text{ VA, Abs}$

-j1Ω: $P = 0$

$Q = -1|\hat{I}_2|^2 = -0.37 \text{ VAR, Abs}$

$\hat{S} = 0.37\angle -90^\circ \text{ VA, Abs}$

j2Ω: $P = 0 \text{ W} \;/\; Q = 2(|\hat{I}_2 - \hat{I}_1|)^2 = 2(3.67)^2 = 26.94 \text{ VAR Abs}$

$\hat{S} = 26.94\angle 90^\circ \text{ VA, Abs}$

(2) cont

$2\hat{V}_o$: need net current, $\hat{I}_o = \hat{I}_2 - \hat{I}_1$

$= 3.67 \angle 154.19^\circ$ Arms

$P = V_{rms} I_{rms} \cos(\theta - \phi)$

$= (|2\hat{V}_o|)(|\hat{I}_o|) \cos(-111.04 - 154.19)$

$= (4.88)(3.67) \cos(-265.23)$

$\boxed{P = -1.46 \text{ W, Del}}$

$Q = (|2\hat{V}_o|)(|\hat{I}_o|) \sin(\theta - \phi)$

$= (4.88)(3.67) \sin(-265.23)$

$\boxed{Q = 17.55 \text{ VAR, Del}}$

$\hat{S} = (2\hat{V}_o)(\hat{I}_o^*) = (4.88 \angle -111.04)(3.67 \angle -154.19)$

$\hat{S} = 17.91 \angle -265.23^\circ$ VA, Del

$8.49 \angle 0^\circ$ Vrms (across $\hat{I}_1$): $P = (8.49)(3.67) \cos(0 - (-16.27))$

$= 29.91$ W, Del

$Q = (8.49)(3.67) \sin(16.27)$

$= 8.73$ VAR, Del

$\hat{S} = (8.49 \angle 0)(3.67 \angle -16.27)^*$

$= (8.49 \angle 0)(3.67 \angle 16.27)$

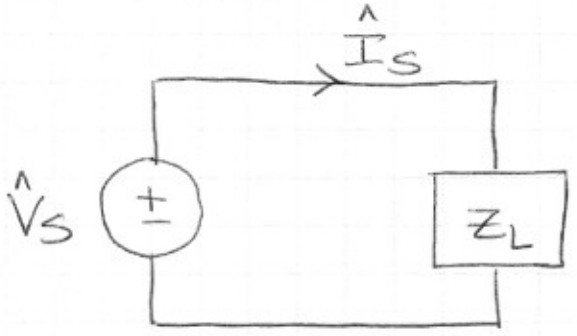
$= 31.16 \angle 16.27^\circ$ VA, del

check P

$\Sigma P_{del} = 29.91 - 1.46 = 28.45$ W checks ☺

$\Sigma P_{abs} = 26.94 + 1.49 = 28.43$ W

(3)



$\hat{V}_S = 220\angle 0^\circ$ Vrms

Z_L: $P_L = 44$ kW
$PF_L = 0.500$ lagging

a) $P_L = V_S I_S \cdot PF_L$

$44 \times 10^3 = 220 \cdot I_S \cdot 0.5$

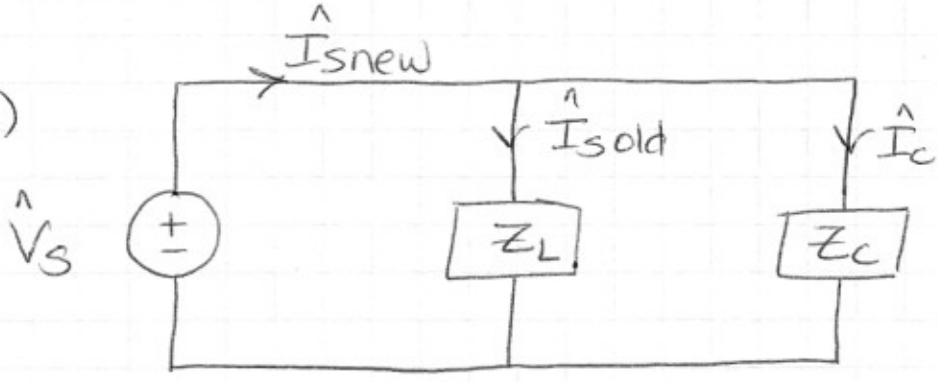
$I_S = 400$ Arms

$\angle Z_L = \cos^{-1}(.5) = 60^\circ$

$\hat{I}_S = 400\angle -60^\circ$ Arms

$Z_L = \dfrac{\hat{V}_S}{\hat{I}_S} = 0.55\angle 60^\circ\ \Omega$

b)



$P_S = P_L = 44$ kW

$PF_{CL} = 0.988$ lag

$\hat{I}_{Sold} = 400\angle -60^\circ$ Arms

$\hat{V}_S = 220\angle 0^\circ$ Vrms

$\angle Z_{CL} = \cos^{-1}(.988) = 8.89^\circ$

$P_S = V_S \cdot I_{Snew} \cdot PF_{CL}$

$44 \times 10^3 = 220 \cdot I_{Snew} \cdot 0.988$

$I_{Snew} = 202.43$ Arms

$\hat{I}_{Snew} = 202.43\angle 8.89^\circ$

$\hat{I}_C = \hat{I}_{Snew} - \hat{I}_{Sold}$

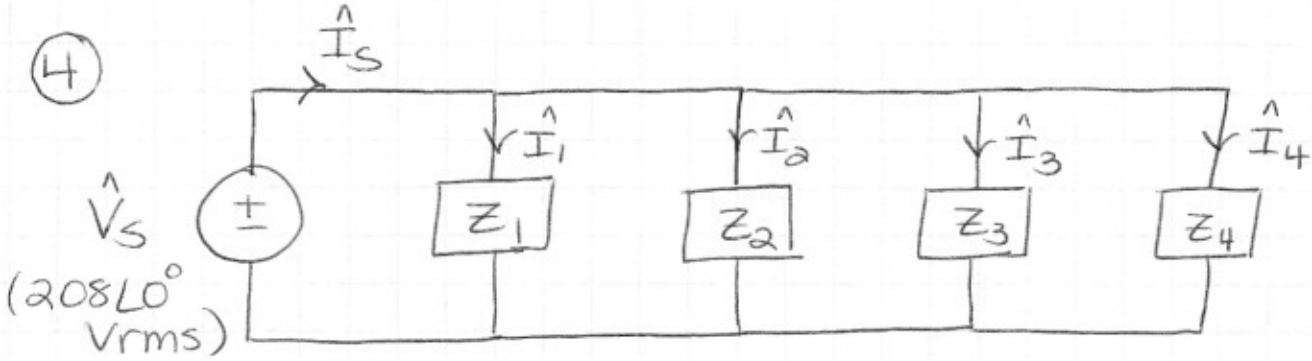
$= 315.13\angle 90^\circ$ Arms

$Z_C = \dfrac{\hat{V}_S}{\hat{I}_C}$

$Z_C = 0.70\angle -90^\circ\ \Omega$

PS#3 solutions

(4)



$P_1 = 18\,kW$, $PF_1 = 0.5$ lag
$|\hat{S}_2| = 10\,kVA$, $PF_2 = 0.2$ lag
$P_3 = 12\,kW$, $PF_3 = 1$
$Q_4 = 16\,kVAR$, $PF_4 = 0.3$ lag

a) Load 1 : $P_1 = V_s I_1 PF_1$

$\angle Z_1 = \cos^{-1}(.5)$ $\quad 18\times10^3 = 208\, I_1 (0.5)$

$\angle Z_1 = 60°$ $\quad I_1 = 173.08$ Arms

$\hat{I}_1 = 173.08\angle -60°$ Arms

$\hat{Z}_1 = \hat{V}_s / \hat{I}_1 = 1.20\angle 60° \,\Omega$

Load 2 : $|\hat{S}_2| = V_s I_2$ $\quad 10\times10^3 = (208) I_2$

$I_2 = 48.08$ Arms

$\angle Z_2 = \cos^{-1}(.2) = 78.46°$

$|Z_2| = \frac{V_s}{I_2} = \frac{208}{48.08} = 4.33\,\Omega$

$\hat{I}_2 = 48.08\angle -78.46°$ Arms

$Z_2 = 4.33\angle 78.46° \,\Omega$

Load 3 : PF = 1 purely resistive load. $\angle Z_3 = 0°$ $\quad P_3 = V_s I_3 \cdot PF_3$

$12\times10^3 = 208\, I_3$

$I_3 = 57.69$ Arms

$|Z_3| = \frac{208}{57.69} = 3.61\,\Omega$

$\hat{I}_3 = 57.69\angle 0°$ Arms

$Z_3 = 3.61\angle 0° \,\Omega$

PS #3 - Solutions

(4) a) cont

LOAD 4: $Q_4 = V_S I_4 \cdot \sin(\angle Z_4)$

$\angle Z_4 = \cos^{-1}(0.3)$

$\angle Z_4 = 72.54^\circ$

$|Z_4| = \frac{V_S}{I_4} = 2.58\,\Omega$

$16 \times 10^3 = 208 \cdot I_4 \cdot \sin(72.54^\circ)$

$I_4 = 80.64$ Arms

$\hat{I}_4 = 80.64 \angle -72.54^\circ$ Arms

$Z_4 = 2.58 \angle 72.54^\circ\,\Omega$

b) We need $|\hat{I}_S|$, first find $\hat{I}_S$.

$\hat{I}_S = \hat{I}_1 + \hat{I}_2 + \hat{I}_3 + \hat{I}_4$ [note: DO NOT ADD ONLY the magnitudes!!!]

$\hat{I}_S = 326.70 \angle -56.98^\circ$ Arms

$|\hat{I}_S|$ or $I_S = 326.70$ Arms

c) method 1 $P_S = V_S I_S \, PF_S$ — PF seen by source or by all loads combined.

$P_S = P_1 + P_2 + P_3 + P_4$

(P_1: given; P_3: given)

$P_2 = V_S I_2 \cdot PF_2 = (208)(48.08)(.2) = 2.00$ kW

-or- $P_2 = |\hat{S}_2| \cdot PF_2 = 10 \times 10^3 (.2) = 2.00$ kW

$P_4 = V_S I_4 \cdot PF_4 = (208)(80.64)(.3) = 5.03$ kW

$P_S = 18 + 2 + 12 + 5.03$

$P_S = 37.03$ kW

$PF_S = \frac{P_S}{V_S I_S} = \frac{37.03 \times 10^3}{(208)(326.70)}$

(4) c) method 1 cont

$\boxed{PF_S = 0.545 \text{ lagging}}$ (all loads are inductive or purely resistive. Combined the loads will be inductive)

method 2

$\hat{S}_S \equiv$ complex power delivered by source

$\hat{S}_S = \hat{V}_S \cdot \hat{I}_S^* = (208\angle 0)(326.70\angle +56.98)$

$\hat{S}_S = 67.95\angle 56.98°$ kVA

the angle of the complex power is the angle of the loads combined.

$\angle Z_{CL} = \angle \hat{S}_S = 56.98$ (inductive $56.98 > 0$)

$PF_S = PF_{CL} = \cos(56.98)$

$\boxed{PF_S = 0.545 \text{ lagging}}$

method 3: $Z_{CL} = \dfrac{\hat{V}_S}{\hat{I}_S} = \dfrac{208\angle 0}{326.70\angle -56.98}$

$Z_{CL} = 0.64\angle 56.98° \Omega$

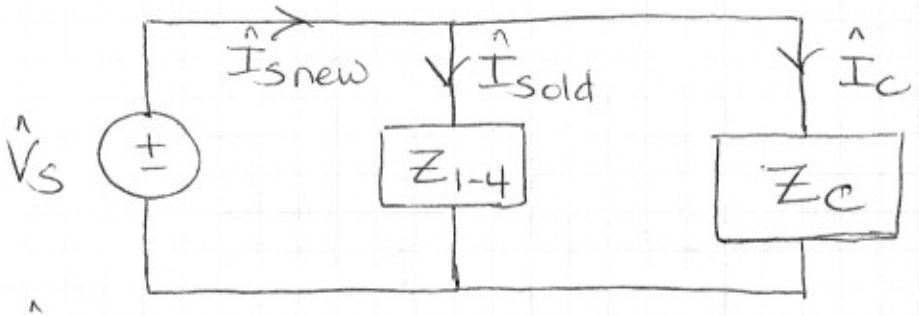
$PF_{CL} = PF_S = \cos(56.98)$

$\boxed{PF_S = 0.545 \text{ lagging}}$

(4) cont

d) $\hat{S}_s = \hat{V}_s \cdot \hat{I}_s^* = 67.95 \angle 56.98 \text{ kVA}$

e) Z_c such that $|\hat{I}_{snew}| = .8|\hat{I}_{sold}|$



(* note we are designing $\hat{I}_{snew}$ & solving for PF_{CL})

$\hat{I}_{sold} = 326.70 \angle -56.98^\circ \text{ Arms}$

$\hat{S}_s = 67.95 \angle 56.98 \text{ kVA}$

$P_s = 37.03 \text{ kW}$ $\qquad \hat{V}_s = 208 \angle 0^\circ \text{ Vrms}$

$|\hat{I}_{snew}| = .8(326.70) = 261.36 \text{ Arms}$

$P_s = V_s I_{snew} \cdot PF_{CL}$

$37.03 \times 10^3 = (208)(261.36) \cdot PF_{CL}$

$PF_{CL} = 0.681$ lagging (we will choose lagging here)

$\angle Z_{CL} = \cos^{-1}(PF_{CL}) = \cos^{-1}(.681)$

$\angle Z_{CL} = 47.08^\circ$

$\hat{I}_{snew} = 261.36 \angle -47.08$

$\hat{I}_c = \hat{I}_{snew} - \hat{I}_{sold}$ (by KCL)

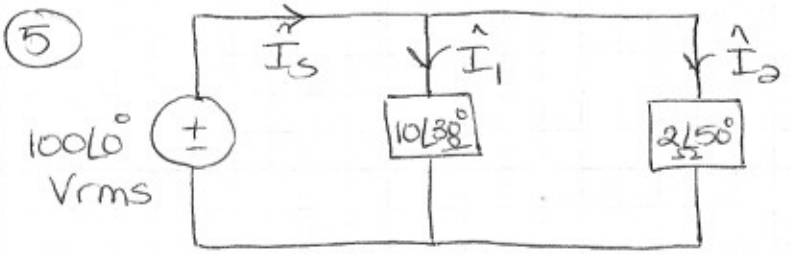
$\hat{I}_c = 82.54 \angle 90^\circ \text{ Arms}$

$$Z_c = \frac{\hat{V}_s}{\hat{I}_c} = 2.52 \angle -90^\circ \Omega$$

STAEDTLER® No. 937 811E Engineer's Computation Pad

STAEDTLER® No. 937 811E Engineer's Computation Pad

(5)



a) $\hat{I}_1 = \frac{100\angle 0}{10\angle 30^\circ} = 10\angle -30^\circ$ Arms

$\hat{I}_2 = \frac{100\angle 0}{2\angle 50} = 50\angle -50^\circ$ Arms

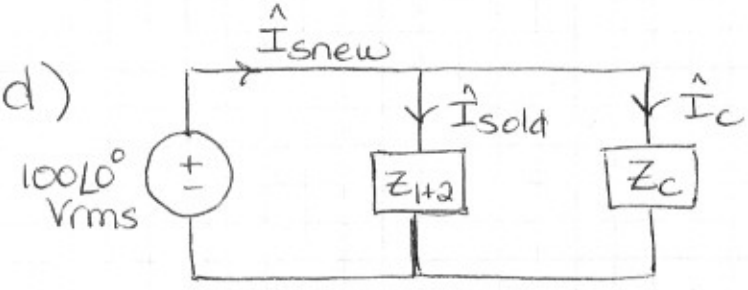
b) $P_1 = (100)(10)\cos(30^\circ) = 866.02$ W

$P_2 = (100)(50)\cos(50^\circ) = 3.21$ kW

c) $Q_1 = 100(10)\sin(30^\circ) = 500$ VAR

$Q_2 = 100(50)\sin(50^\circ) = 3.83$ kVAR

d)



$\hat{I}_{sold} = \hat{I}_1 + \hat{I}_2$

$= 59.50\angle -46.7^\circ$ Arms

$P_S = P_1 + P_2$

$P_S = 4.08$ kW

$PF_{CL} = 0.9$ lag

$\angle Z_{CL} = \cos^{-1}(.9) = 25.84^\circ$

$4.08 \times 10^3 = 100(I_{snew})(.9)$

$I_{snew} = 45.29$ Arms

$\hat{I}_{snew} = 45.29\angle -25.84^\circ$ Arms

$\hat{I}_c = 23.56\angle 90^\circ$ Arms

$Z_c = \frac{100\angle 0}{\hat{I}_c} = 4.24\angle -90^\circ\ \Omega$

$\frac{1}{\omega C} = 4.24$

$\omega = 377$ rad/s

$C = 0.63$ mF