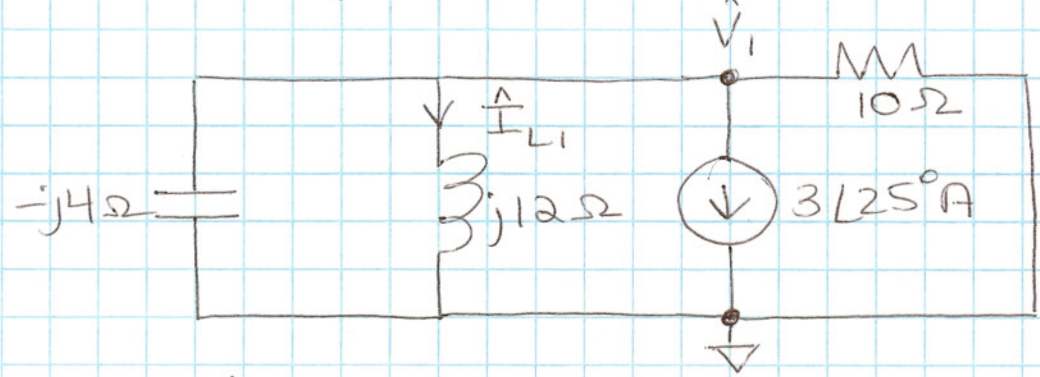
Problem Set #2 - solutions

(1) superposition

$3\angle 25^\circ$ A on



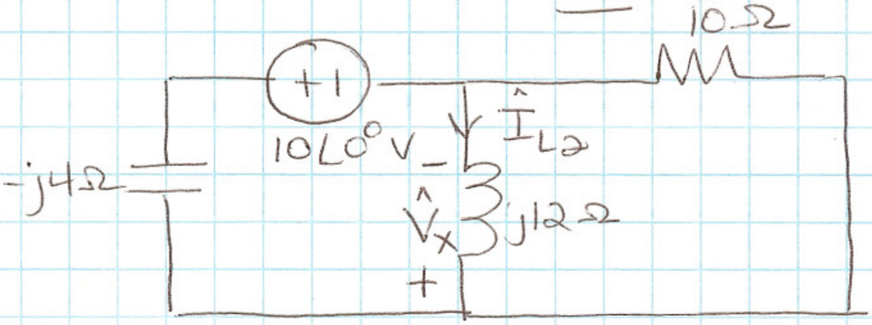
$$\hat{I}_{L1} = \frac{\hat{V}_1}{j12}$$

$$\frac{\hat{V}_1}{-j4} + \frac{\hat{V}_1}{j12} + \frac{\hat{V}_1}{10} + 3\angle 25 = 0$$

$$\hat{V}_1 = 15.43\angle 145.96^\circ V$$

$$\boxed{\hat{I}_{L1} = 1.28\angle 55.96^\circ A}$$

$10\angle 0^\circ$ V ON



by voltage division

$$\hat{V}_x = \frac{10\angle 0 \cdot (j12 \| 10)}{-j4 + (j12 \| 10)}$$

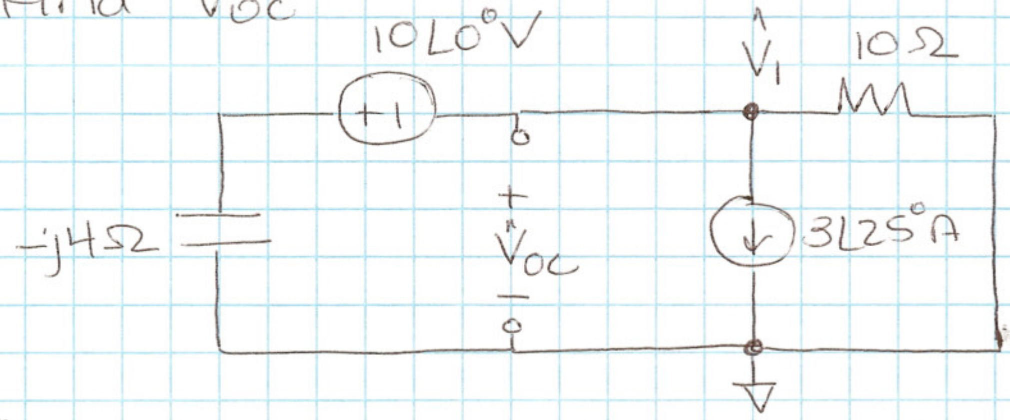
$$= 12.86\angle 30.96^\circ V$$

$$\hat{I}_{L2} = \frac{-\hat{V}_x}{j12} = 1.07\angle 120.96^\circ A$$

$$\hat{I}_{L1} + \hat{I}_{L2} = \hat{I}_L = 1.99\angle 85.15^\circ A$$

① cont using Thev. Equiv. Ckt.

Find $\hat{V}_{OC}$

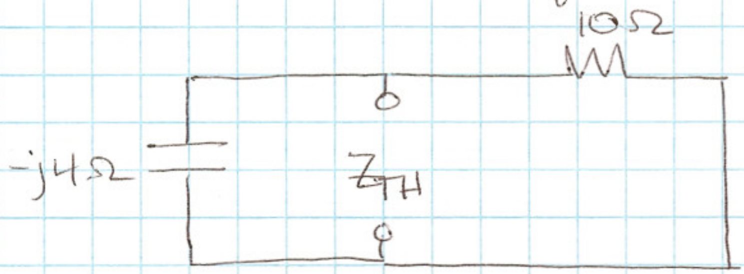


$\hat{V}_1 = \hat{V}_{OC}$

$$\frac{\hat{V}_1 + 10\angle 0}{-j4} + (3\angle 25) + \frac{\hat{V}_1}{10} = 0 \qquad \hat{V}_1 = 17.26\angle 165.99^\circ\ V$$

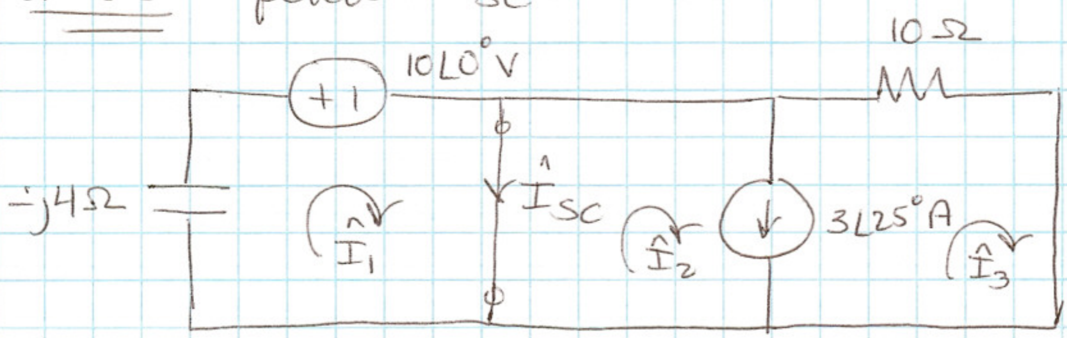
$$\hat{V}_{TH} = \hat{V}_{OC} = 17.26\angle 165.99^\circ\ V$$

Find Z_{TH} (use quick method)



$Z_{TH} = 10 \| -j4 = 3.71\angle -68.20^\circ\ \Omega$

check find $\hat{I}_{SC}$



Know: $\hat{I}_3 = 0$

$\hat{I}_2 - \hat{I}_3 = 3\angle 25^\circ$

$\hat{I}_2 = 3\angle 25^\circ$

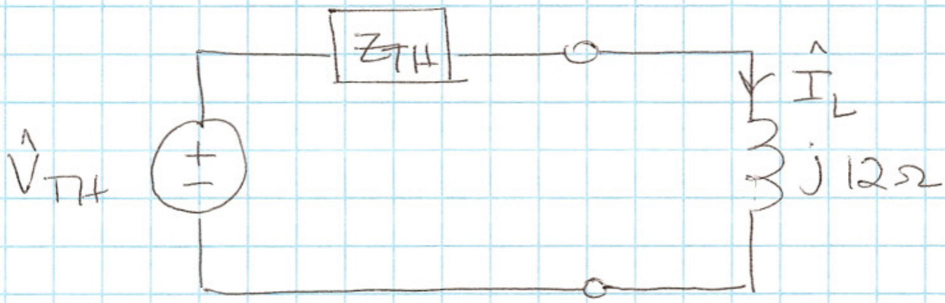
$\hat{I}_{SC} = \hat{I}_1 - \hat{I}_2$

m1: $-(10\angle 0) - (-j4)\hat{I}_1 = 0 \qquad \hat{I}_1 = 2.5\angle 90^\circ$

$\hat{I}_{SC} = 4.65\angle -125.81\ A \qquad \frac{\hat{V}_{OC}}{\hat{I}_{SC}} = 3.71\angle -68.20^\circ\ \Omega$

checks

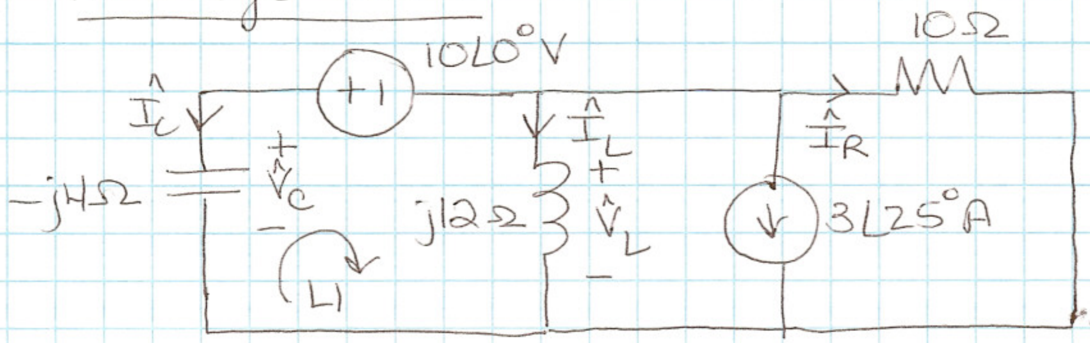
① cont



$$\hat{I}_L = \frac{\hat{V}_{TH}}{Z_{TH} + j12} = 1.99\angle 85.14^\circ \text{ A}$$

checks w/ superpositi

Average Power



for $\hat{I}_L = 1.99\angle 85.15^\circ$ A $\quad \hat{V}_L = j12\hat{I}_L = 23.91\angle 175.15^\circ$ V

by KVL at loop 1 : $\hat{V}_C - 10\angle 0 - \hat{V}_L = 0$ $\quad \boxed{\hat{V}_C = 13.97\angle 171.67}$

$\hat{I}_R = \hat{V}_L/10 = 2.39\angle 175.15^\circ$ A $\quad \hat{I}_C = \dfrac{\hat{V}_C}{-j4} = 3.49\angle -98.32$

element	Paverage	Del / Abs
$-j4\Omega$	0 W	—
$j12\Omega$	0 W	—
10Ω	$\frac{1}{2}(2.39)^2(10) = 28.56$ W	Abs
$10\angle 0^\circ$ V	$P = \frac{1}{2}(10)(3.49)\cos(0-(-98.32))$ $= -2.53$ W	Del
$3\angle 25^\circ$ A	$P = \frac{1}{2}(23.91)(3)\cos(175.15^\circ - 25)$ $= -31.11$ W	Abs

① cont

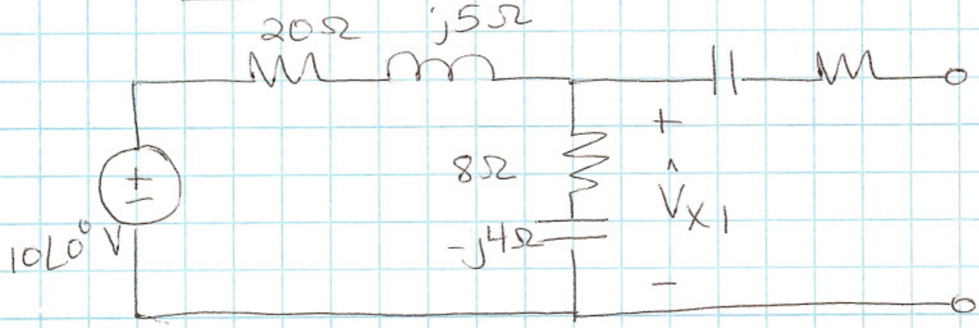
$\Sigma P_{del} = -2.53\ W$

$\Sigma P_{abs} = 28.56 + (-31.11) = -2.54\ W$

agrees!

② superposition

$\underline{10\angle 0^\circ}$ $\underline{ON}$

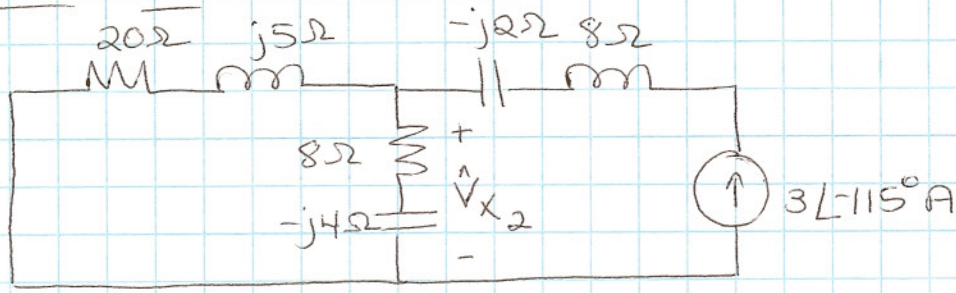


$\underline{\text{by voltage division}}$

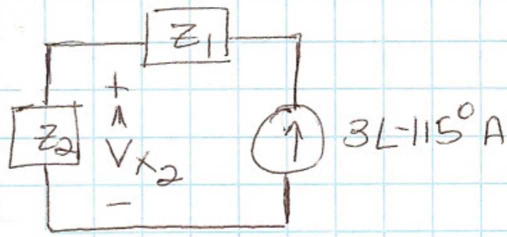
$$\hat{V}_{x_1} = 10\angle 0 \frac{(8-j4)}{(8-j4)+(20+j5)}$$

$$\boxed{\hat{V}_{x_1} = 3.19\angle -28.61^\circ\ V}$$

$\underline{3\angle -115^\circ}$ $\underline{ON}$



simplify



$Z_1 = 8 - j2\ \Omega$

$Z_2 = (20+j5) \| (8-j4)$

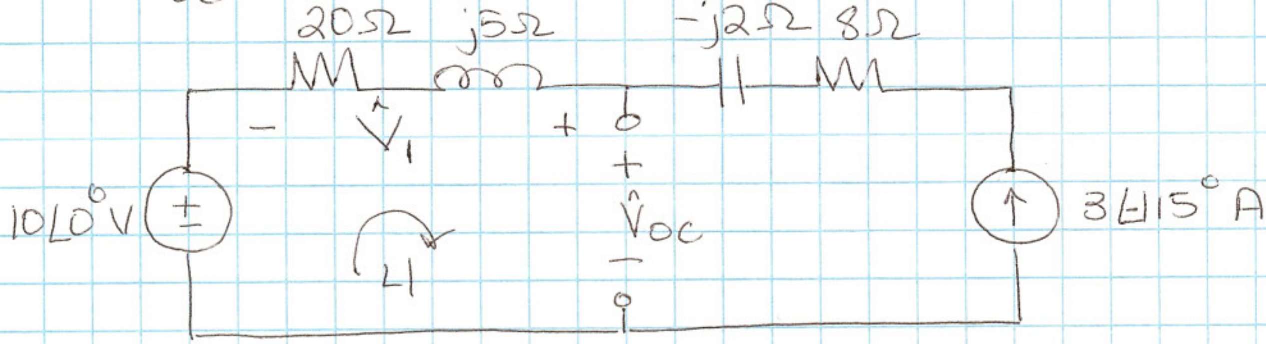
$= 6.58\angle -14.57^\circ\ \Omega$

$$\hat{V}_{x_2} = (3\angle -115^\circ)(Z_2)$$
$$= 19.74\angle -129.57^\circ\ V$$

$$\boxed{\hat{V}_x = \hat{V}_{x_1} + \hat{V}_{x_2} = 19.39\angle -120.27^\circ\ V}$$

② cont : Thev.

Find $\hat{V}_{OC}$

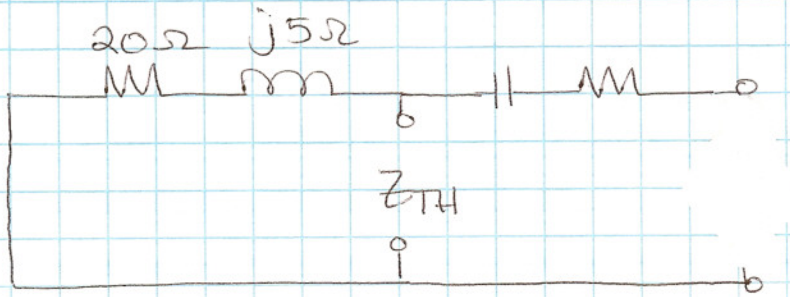


$$\hat{V}_1 = (3\angle{-115})(20+j5) = 61.85\angle{-100.96^\circ}\ V$$

KVL at L1 : $10\angle 0 + \hat{V}_1 - \hat{V}_{OC} = 0$

$$\hat{V}_{OC} = 60.74\ \angle{-91.66^\circ}\ V$$

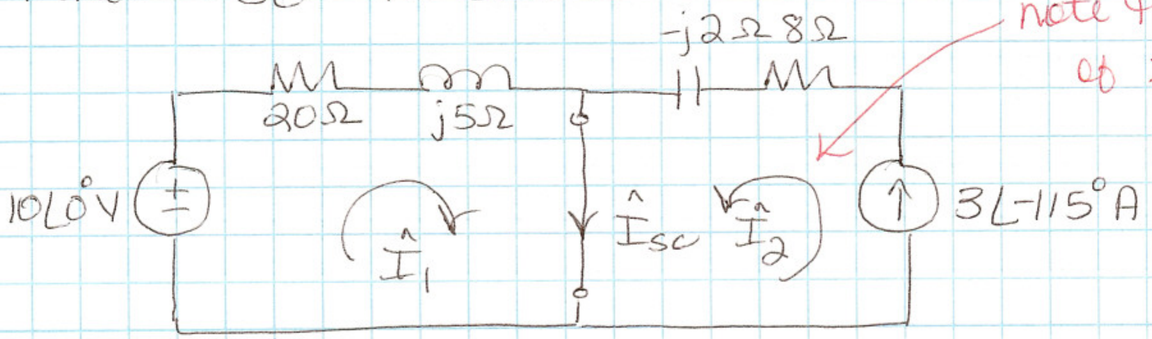
$\underline{Z_{TH}}$



$$Z_{TH} = 20 + j5\ \Omega \quad \text{or} \quad 20.62\angle 14.04^\circ\ \Omega$$

Find $\hat{I}_{SC}$ to check

note the direction of the mesh current



$\hat{I}_2 = 3\angle{-115^\circ}\ A$

$\hat{I}_{SC} = \hat{I}_1 + \hat{I}_2$

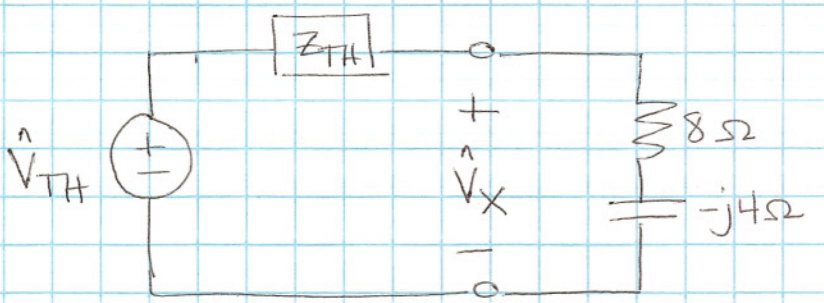
m1 : $10\angle 0 - (20+j5)\hat{I}_1 = 0$

$\hat{I}_1 = 0.485\angle{-14.04^\circ}\ A$

$$\boxed{\hat{I}_{SC} = 2.95\angle{-105.70^\circ}\ A}$$

$$Z_{TH} = \frac{\hat{V}_{OC}}{\hat{I}_{SC}} = 20.59\angle 14.04$$ ☺ checks

Equivalent ckt

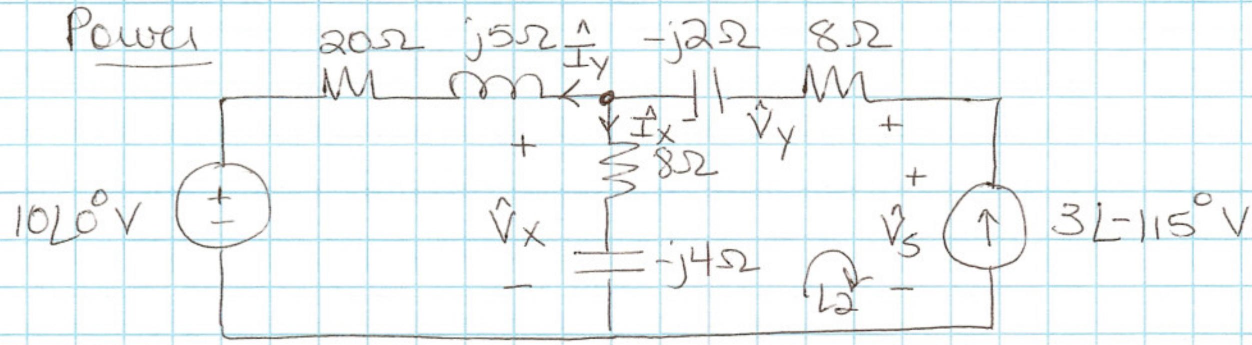


$$\hat{V}_X = \hat{V}_{TH} \cdot \frac{8-j4}{(8-j4)+Z_{TH}}$$

$$\boxed{\hat{V}_X = 19.39\angle{-120.27^\circ}\ V}$$

checks w/ superposition

Power



$\hat{V}_x = 19.39\angle{-120.27^\circ}\ V \qquad \hat{I}_x = \frac{\hat{V}_x}{8-j4} = 2.17\angle{-93.70^\circ}\ A$

by KCL: $3\angle{-115^\circ} = \hat{I}_x + \hat{I}_y$

$\hat{I}_y = 1.26\angle{-153.77^\circ}\ A$

$\hat{V}_y = (3\angle{-115})(8-j2) = 24.74\angle{-129.04^\circ}\ V$

by KVL (L1) $\hat{V}_x + \hat{V}_y - \hat{V}_S = 0$ $\qquad \boxed{\hat{V}_S = 44.00\angle{-125.15^\circ}\ V}$

element	Power (W)	Del/Abs
j5Ω	0	—
−j2Ω	0	—
−j4Ω	0	—
20Ω	$P = \frac{1}{2}(20)(1.26)^2 = 15.88\ W$	Abs
8Ω (middle)	$P = \frac{1}{2}(8)(2.17)^2 = 18.84\ W$	Abs
8Ω (right)	$P = \frac{1}{2}(8)(3)^2 = 36\ W$	Abs
$10\angle 0^\circ V$	$P = \frac{1}{2}(10)(1.26)\cos(0+153.77)$	

$P = -5.65\ W$ abs OR $P = 5.65\ W$ Del

(2)

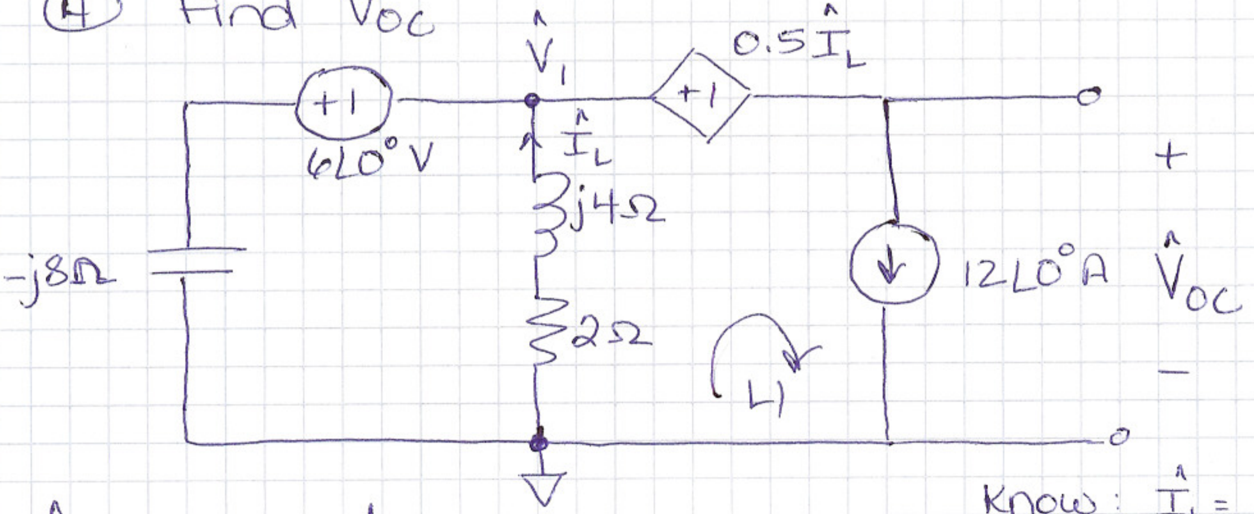
element	Power	Del / Abs
$3\angle -115^\circ$ A	$P = \frac{1}{2}(44)(3)\cos(-125.14+115)$	
	$= 64.69$ W, del	

$\Sigma P_{del} = 64.69 + 5.65 = 70.62\,W$

$\Sigma P_{abs} = 15.88 + 18.84 + 36 = 70.72\,W$

(3) see work on Probs 1 & 2.

(4) Find $\hat{V}_{OC}$



Know: $\hat{I}_L = \frac{-\hat{V}_1}{2+j4}$

$$\frac{\hat{V}_1 + 6\angle 0}{-j8} + \frac{\hat{V}_1}{2+j4} + 12\angle 0 = 0$$

$$\hat{V}_1\left(\frac{1}{-j8} + \frac{1}{2+j4}\right) = \frac{6\angle 0}{j8} - 12\angle 0$$

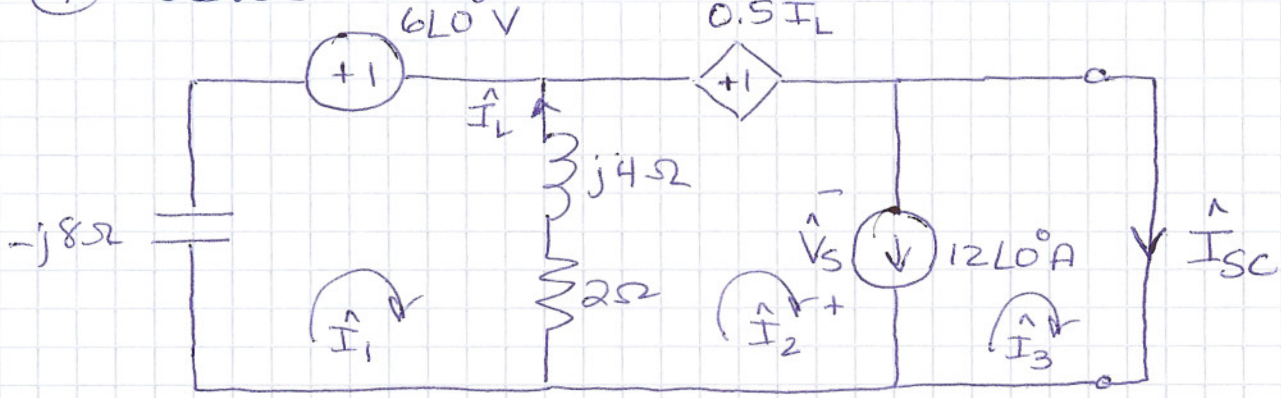
$$\hat{V}_1 = 96.19\angle -139.55^\circ\ V$$

$$\hat{I}_L = 21.51\angle -22.99^\circ\ A$$

By KVL around L1: $\hat{V}_1 - 0.5\hat{I}_L - \hat{V}_{OC} = 0$

$$\hat{V}_{OC} = 101.45\angle -144.99^\circ\ V$$

(4) cont



Know $\hat{I}_L = \hat{I}_2 - \hat{I}_1$ $\quad \hat{I}_2 - \hat{I}_3 = 12\angle 0^\circ$

$\hat{V}_S = 0$ $\quad \hat{I}_3 = \hat{I}_{SC}$

m1: $-(-j8)\hat{I}_1 - 6\angle 0 - (2+j4)(\hat{I}_1 - \hat{I}_2) = 0$

m2: $-(2+j4)(\hat{I}_2 - \hat{I}_1) - 0.5\hat{I}_L + \hat{V}_S = 0$

m1 $\boxed{\hat{I}_1(-2+j4) + \hat{I}_2(2+j4) = 6\angle 0}$

m2 $-(2+j4)(\hat{I}_2 - \hat{I}_1) - 0.5(\hat{I}_2 - \hat{I}_1) = 0$

$\hat{I}_1(2.5+j4) + \hat{I}_2(-2.5-j4) = 0$

or $\boxed{\hat{I}_1 = \hat{I}_2}$

so: $\hat{I}_2(-2+j4) + \hat{I}_2(2+j4) = 6\angle 0$

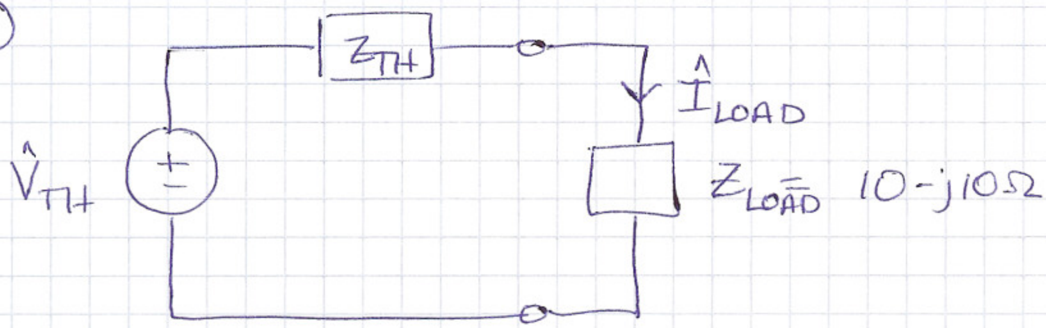
$\hat{I}_2 = 0.75\angle -90^\circ$ A

$\hat{I}_3 = \hat{I}_2 - 12\angle 0 = 12.02\angle -176.42^\circ$ A

$\boxed{\hat{I}_{SC} = 12.02\angle -176.42^\circ \text{ A}}$

$Z_{TH} = \dfrac{\hat{V}_{OC}}{\hat{I}_{SC}} = 8.44\angle 31.42^\circ$

4

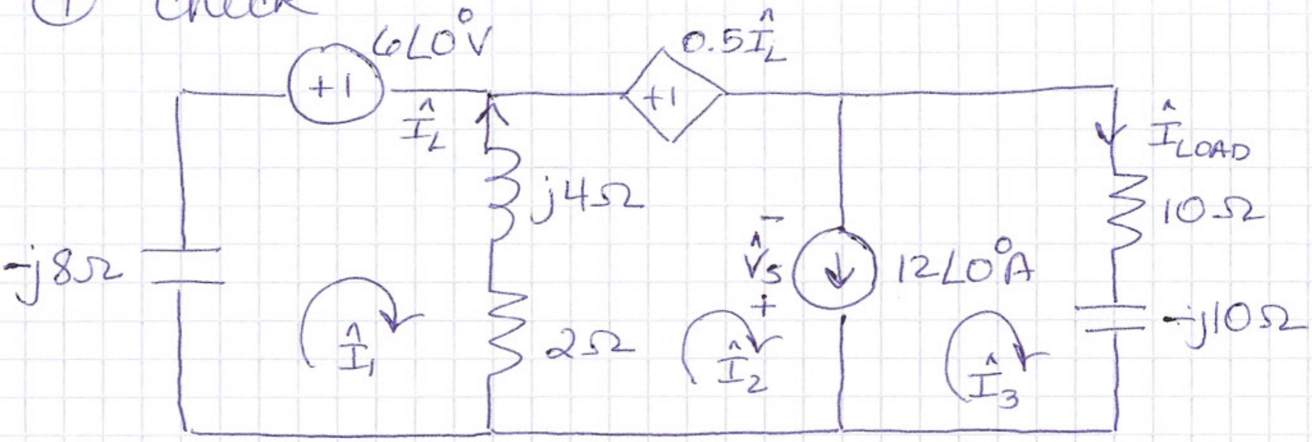


$$\hat{I}_{LOAD} = \frac{\hat{V}_{TH}}{Z_{TH} + Z_{LOAD}} = 5.61 \angle -126.96^\circ \text{ A}$$

$$P_{LOAD} = \frac{1}{2}(10)(5.61)^2$$

$$= 157.36 \text{ W, abs}$$

④ check



Know: $\hat{I}_L = \hat{I}_2 - \hat{I}_1$ $\quad \hat{I}_{LOAD} = \hat{I}_3$

$12\angle 0 = \hat{I}_2 - \hat{I}_3$

m1: $-(-j8)\hat{I}_1 - 6\angle 0 - (2+j4)(\hat{I}_1 - \hat{I}_2) = 0$

m2: $-(2+j4)(\hat{I}_2 - \hat{I}_1) - 0.5\hat{I}_L + \hat{V}_S = 0$

m3: $-\hat{V}_S - (10-j10)\hat{I}_3 = 0$

$\boxed{\text{m1: } \hat{I}_1(-2+j4) + \hat{I}_2(2+j4) = 6\angle 0}$

add m2 & m3 $-(+2+j4)(\hat{I}_2 - \hat{I}_1) - .5(\hat{I}_2 - \hat{I}_1) - (10-j10)\hat{I}_3 = 0$

$\boxed{\hat{I}_1(2.5+j4) + \hat{I}_2(-2.5-j4) + \hat{I}_3(-10+j10) = 0}$

$\boxed{\hat{I}_2 - \hat{I}_3 = 12\angle 0}$

$$\begin{bmatrix} -2+j4 & 2+j4 & 0 \\ 2.5+j4 & -2.5-j4 & -10+j10 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \\ \hat{I}_3 \end{bmatrix} = \begin{bmatrix} 6\angle 0 \\ 0 \\ 12\angle 0 \end{bmatrix}$$

$\hat{I}_1 = 8.67\angle 104.63° A$

$\hat{I}_2 = 9.72\angle -27.45° A$

$\hat{I}_3 = 5.61\angle -126.95° A$

$\hat{I}_L = 16.81\angle -49.96° A$

$\hat{V}_S = 79.34\angle 8.05° V$

$\Sigma P_{abs} =$

$P_{LOAD} = \frac{1}{2}(10)(5.61)^2 = 157.36$ W, abs

2Ω: $P = \frac{1}{2}(2)(16.81)^2 =$ 282.58 W, abs

$6\angle 0$: $P = \frac{1}{2}(6)(8.67)\cos(0 - 104.63) = -6.57$ W, abs

$12\angle 0$: $P = \frac{1}{2}(79.34)(12)\cos(8.05 - 0) = 471.35$ W, del

$0.5\hat{I}_L$: $P = \frac{1}{2}(8.40)(9.72)\cos(-49.96 + 27.45°) = 37.71$ W, abs