

Problem Set 1 – Solutions

1.

① a) $v_1(t) = 4 \cos 5t \text{ V}$
 $v_2(t) = 10 \cos(5t - 20^\circ) \text{ V}$
 $\text{angle} = (0 - (-20)) = 20^\circ$

v_1 leads v_2 by 20°

b) $v_x(t) = -10 \cos(100t + 50^\circ) \text{ V}$
 $= 10 \cos(100t - 130^\circ) \text{ V}$

v_y leads v_x by 50°

$v_y(t) = 10 \sin(100t + 10^\circ) \text{ V}$
 $= 10 \cos(100t - 80^\circ) \text{ V}$

$-80 - (-130) = 50^\circ$

c) $i_x(t) = -2 \sin(t - 130^\circ) \text{ A}$
 $= 2 \sin(t + 50^\circ) \text{ A}$
 $= 2 \cos(t - 40^\circ) \text{ A}$

i_x leads i_y by 40°

$i_y(t) = -5 \cos(t + 100^\circ) \text{ A}$
 $= 5 \cos(t - 80^\circ) \text{ A}$

$-40 - (-80) = 40^\circ$

2.

a) $v(t) = 10 \cos(120t - 225^\circ) \text{ V}$
 $= 10 \cos(120t + 135^\circ) \text{ V}$

$v(t) = 10 \cos(120t + 135^\circ) + j 10 \sin(120t + 135^\circ) \text{ V}$

$\times v(t) = 10 e^{j(120t + 135^\circ)} \text{ V}$

$\times \hat{V} = 10 \angle 135^\circ \text{ V}$

b) $i(t) = 5 \sin(600t - 125^\circ) \text{ A}$
 $= 5 \cos(600t - 215^\circ) \text{ A}$
 $= 5 \cos(600t + 145^\circ) \text{ A}$

$i(t) = 5 \cos(600t + 145^\circ) + j 5 \sin(600t + 145^\circ) \text{ A}$

$\times i(t) = 5 e^{j(600t + 145^\circ)} \quad \times \hat{I} = 5 \angle 145^\circ \text{ A}$

Problem Set 1 – Solutions

2. continued

$$\begin{aligned} c) \quad v(t) &= -3 \sin(20t) \text{ V} \\ &= 3 \sin(20t + 180^\circ) \text{ V} \\ &= 3 \cos(20t + 90^\circ) \text{ V} \end{aligned}$$

$$\begin{aligned} v(t) &= 3 \cos(20t + 90^\circ) + j3 \sin(20t + 90^\circ) \text{ V} \\ &= 3e^{j(20t + 90^\circ)} \text{ V} \end{aligned}$$

$$\hat{V} = 3 \angle 90^\circ \text{ V}$$

$$\begin{aligned} d) \quad i(t) &= -10 \cos(2t + 45^\circ) \text{ A} \\ &= 10 \cos(2t - 135^\circ) \text{ A} \end{aligned}$$

$$i(t) = 10 \cos(2t - 135^\circ) + j10 \sin(2t - 135^\circ) \text{ A}$$

$$i(t) = 10e^{j(2t - 135^\circ)}$$

$$\hat{I} = 10 \angle -135^\circ \text{ A}$$

3.

$$(3) \quad \omega = 5 \text{ rad/s}$$

$$a) \quad 6 \angle 25^\circ + 10 \angle -40^\circ$$

$$(5.44 + j2.54)$$

$$+ (7.66 - j6.43)$$

$$= 13.10 - j3.89$$

$$\sqrt{(13.10)^2 + (-3.89)^2} = 13.66$$

$$\tan^{-1}\left(\frac{-3.89}{13.10}\right) = -16.55^\circ$$

$$13.66 \angle -16.55^\circ \Rightarrow 13.66 \cos(5t - 16.55^\circ) + j13.66 \sin(5t - 16.55^\circ)$$

$$6 \angle 25^\circ = 6 \cos 25^\circ + j6 \sin 25^\circ$$

$$= 5.44 + j2.54$$

$$(10 \angle -40^\circ) = 10 \cos(-40^\circ) + j10 \sin(-40^\circ)$$

$$= 7.66 - j6.43$$

Problem Set 1 – Solutions

3. continued

$$\begin{aligned}
 & \text{b) } (5 \angle 80^\circ)(2+j4) \\
 & (5 \angle 80^\circ)(4.47 \angle 63.43^\circ) \\
 & 5(4.47) \angle 80+63.43 \\
 & \boxed{22.36 \angle 143.43} \Rightarrow 22.36 \cos(5t + 143.43) + j22.36 \sin(5t + 143)
 \end{aligned}$$

$2+j4$
 $\sqrt{2^2+4^2} = 4.47$
 $\tan^{-1}(\frac{4}{2}) = 63.43^\circ$

$$\begin{aligned}
 \text{c) } (-1-j8) + (6-j5) &= 5-j13 \\
 &= 13.93 \angle -68.96^\circ \\
 &13.93 \cos(5t - 68.96) + j13.93 \sin(5t - 68.96)
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } (2 \angle 140^\circ) + (3-j6) &= (-1.53 + j1.29) + (3-j6) \\
 &= 1.47 - j4.71 \\
 &= 4.94 \angle -72.71^\circ
 \end{aligned}$$

$$4.94 \cos(5t - 72.71) + j4.94 \sin(5t - 72.71^\circ)$$

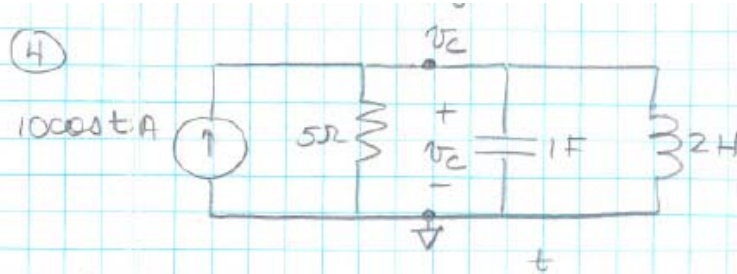
$$\begin{aligned}
 \text{e) } \frac{(-4-j3)}{2 \angle 10} &= \frac{(5 \angle -143.13^\circ)}{2 \angle 10} = 2.5 \angle -153.13^\circ \\
 &2.5 \cos(5t - 153.13^\circ) + j2.5 \sin(5t - 153.13^\circ)
 \end{aligned}$$

$$\text{f) } \frac{10 \angle -25^\circ}{-2+j10} = \frac{10 \angle -25}{10.20 \angle 101.31} = 0.98 \angle -126.31^\circ$$

$$0.98 \cos(5t - 126.31) + j0.98 \sin(5t - 126.31)$$

Problem Set 1 – Solutions

4.



$$\frac{d}{dt} \left(\frac{v_c}{5} + 1 \frac{dv_c}{dt} + \frac{1}{2} \int_0^t v_c dt = 10 \cos t \right)$$

$$\frac{d^2 v_c}{dt^2} + \frac{1}{5} \frac{dv_c}{dt} + \frac{1}{2} v_c = -10 \sin t$$

$$v_c(t) = A \cos t + B \sin t \quad V$$

$$\frac{dv_c}{dt} = -A \sin t + B \cos t$$

$$\frac{d^2 v_c}{dt^2} = -A \cos t - B \sin t$$

$$(-A \cos t - B \sin t) + \frac{1}{5}(-A \sin t + B \cos t) + \frac{1}{2}(A \cos t + B \sin t) = -10 \sin t$$

$$\cos t \left(-A + \frac{B}{5} + \frac{1}{2}A \right) + \sin t \left(-B - \frac{A}{5} + \frac{1}{2}B \right) =$$

$$-0.5A + 0.2B = 0$$

$$-0.2A - 0.5B = -10$$

$$-10 \sin t$$

$$A = 6.90 \text{ V} \quad B = 17.24 \text{ V}$$

$$v_c(t) = 6.90 \cos t + 17.24 \sin t$$

$$\text{or } v_c(t) = 18.57 \cos(t - 68.2^\circ) \text{ V}$$

Problem Set 1 - Solutions

4. continued

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(14) cont

$$\frac{d^2 v_c}{dt^2} + \frac{1}{5} \frac{dv_c}{dt} + \frac{1}{2} v_c = j10e^{jt}$$

$$\begin{aligned} \text{note: } 10\cos t + j10\sin t &= 10e^{jt} \\ \frac{d(10e^{jt})}{dt} &= j10e^{jt} \end{aligned}$$

$$\begin{aligned} v_c(t) &= Ae^{jt} \\ \frac{dv_c}{dt} &= jAe^{jt} \end{aligned}$$

$$-Ae^{jt} + \frac{j}{5} Ae^{jt} + \frac{A}{2} e^{jt} = j10e^{jt}$$

$$\frac{dv_c}{dt^2} = -Ae^{jt}$$

cancel e^{jt} terms

$$-A + \frac{j}{5} A + \frac{A}{2} = j10$$

$$A(-.5 + j.2) = j10$$

$$A = \frac{j10}{-.5 + j.2} = 18.57 \angle -68.2^\circ$$

$$\begin{aligned} v_c(t) &= 18.57 \angle -68.2^\circ e^{jt} \text{ V} \\ &= 18.57 e^{-j68.2^\circ} e^{jt} \text{ V} \\ &= 18.57 e^{j(t-68.2^\circ)} \text{ V} \end{aligned}$$

$$= 18.57 \cos(t-68.2^\circ) + j18.57 \sin(t-68.2^\circ)$$

suppress imaginary part

$$v_c(t) = 18.57 \cos(t-68.2^\circ) \text{ V}$$

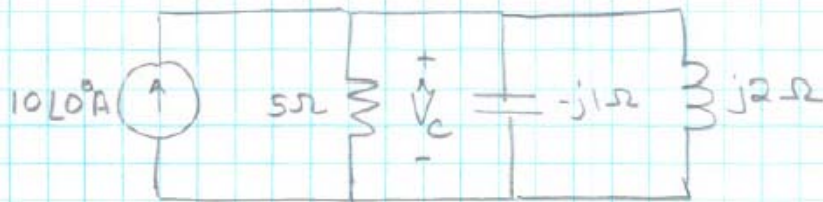
checks w/ part (a) ☺

Problem Set 1 – Solutions

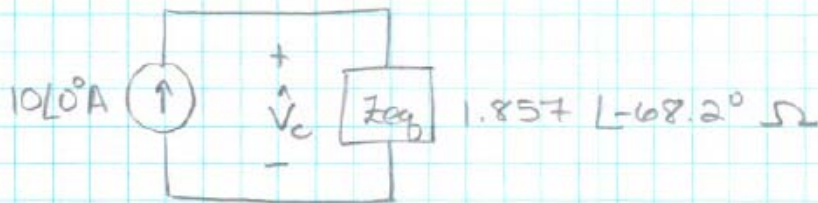
4. continued

④ check using Frequency Domain Analysis

$$\begin{array}{lll} 10 \cos t \text{ A} & \rightarrow & 10 \angle 0^\circ \text{ A} \\ 5 \Omega & \rightarrow & 5 \Omega \\ 1 \text{ F} & \rightarrow & -j1 \Omega \\ 2 \text{ H} & \rightarrow & j2 \Omega \end{array} \quad \omega = 1 \text{ rad/s}$$



$$Z_{eq} = 5 \Omega \parallel -j1 \parallel j2 = 1.857 \angle -68.2^\circ \Omega$$



$$\hat{V}_c = (10 \angle 0^\circ) Z_{eq} = 18.57 \angle -68.2^\circ \text{ V}$$

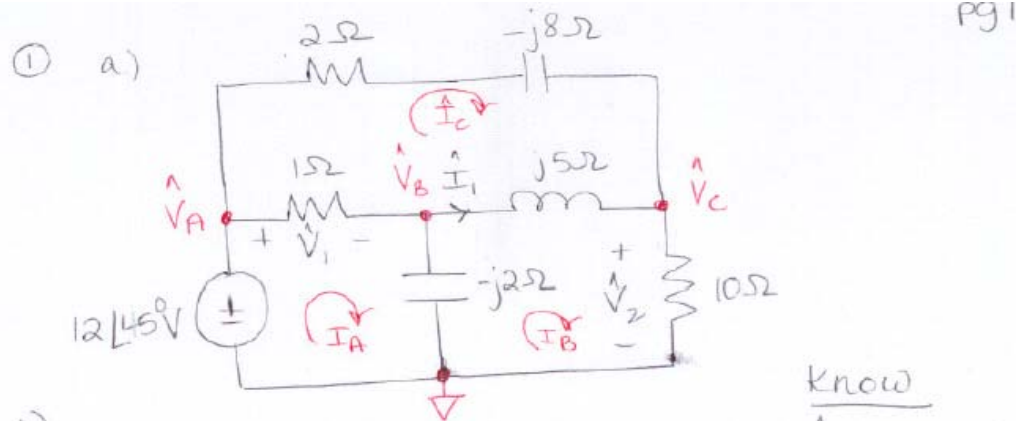
$$\text{so } v_c(t) = 18.57 \cos(t - 68.2^\circ) \text{ V}$$

checks w/ parts a + b



Problem Set 1 - Solutions

5.



know
 $\hat{V}_A = 12 \angle 45^\circ \text{ V}$

b)
 Node A: don't need

Node B: $\frac{\hat{V}_B - \hat{V}_A}{1} + \frac{\hat{V}_B}{-j2} + \frac{\hat{V}_B - \hat{V}_C}{j5} = 0$

Node C: $\frac{\hat{V}_C - \hat{V}_B}{j5} + \frac{\hat{V}_C - \hat{V}_A}{2 - j8} + \frac{\hat{V}_C}{10} = 0$

$\hat{V}_2 = \hat{V}_C \quad \hat{V}_1 = \hat{V}_A - \hat{V}_B \quad \hat{I}_1 = \frac{\hat{V}_B - \hat{V}_C}{j5\Omega}$

simplify

Node B: $\hat{V}_B (1.04 \angle 16.7^\circ) + \hat{V}_C (.2 \angle 90^\circ) = 12 \angle 45^\circ$

Node C: $\hat{V}_B (.2 \angle 90^\circ) + \hat{V}_C (0.15 \angle -32.47^\circ) = 1.46 \angle 120.96^\circ$

Solve $\hat{V}_B = 10.67 \angle 27.65^\circ \text{ V}$

$\hat{V}_C = 4.55 \angle -36.99^\circ \text{ V}$

$\hat{V}_1 = 3.66 \angle 105.29^\circ \text{ V}$

$\hat{V}_2 = 4.55 \angle -36.99^\circ \text{ V}$

$\hat{I}_1 = 1.93 \angle -37.11^\circ \text{ A}$

Problem Set 1 – Solutions

5. continued

mesh (see original circuit for mesh labels)

$$\text{mesh A: } 12 \angle 45^\circ - 1(\hat{I}_A - \hat{I}_C) - (j2)(\hat{I}_A - \hat{I}_B) = 0$$

$$\text{mesh B: } -(j2)(\hat{I}_B - \hat{I}_A) - j5(\hat{I}_B - \hat{I}_C) - 10\hat{I}_B = 0$$

$$\text{mesh C: } -(2-j8)\hat{I}_C - j5(\hat{I}_C - \hat{I}_B) - 1(\hat{I}_C - \hat{I}_A) = 0$$

$$\hat{I}_1 = \hat{I}_B - \hat{I}_C \quad \hat{V}_1 = 1(\hat{I}_A - \hat{I}_C) \quad \hat{V}_2 = 10\hat{I}_B$$

simplify

$$\text{mesh A: } \hat{I}_A(-1+j2) + \hat{I}_B(-j2) + \hat{I}_C(1) = 12 \angle -135^\circ$$

$$\text{mesh B: } \hat{I}_A(-j2) + \hat{I}_B(-10-j3) + \hat{I}_C(j5) = 0$$

$$\text{mesh C: } \hat{I}_A(1) + \hat{I}_B(j5) + \hat{I}_C(-3+j3) = 0$$

$$\hat{I}_A = 4.93 \angle 115.44^\circ \text{ A}$$

$$\hat{I}_B = 0.445 \angle -36.94^\circ \text{ A}$$

$$\hat{I}_C = 1.48 \angle +142.13^\circ \text{ A}$$

$$\hat{I}_1 = 1.93 \angle -37.66^\circ \text{ A}$$

$$\hat{V}_1 = 4.45 \angle -36.94^\circ \text{ V}$$

$$\hat{V}_2 = 3.67 \angle 105^\circ \text{ V}$$

$$i_1(t) = 1.93 \cos(t - 37.66^\circ) \text{ A}$$

$$v_1(t) = 4.45 \cos(t - 36.94^\circ) \text{ V}$$

$$v_2(t) = 3.67 \cos(t + 105^\circ) \text{ V}$$

Problem Set 1 – Solutions

6.

②

note I changed to \hat{V}_A

Node 1: $2 \angle 30^\circ + \frac{\hat{V}_1}{-j2} + \hat{I}_y = 0$

Node 2: $-\hat{I}_y + \frac{\hat{V}_2 - 5\hat{I}_x}{j8} + \frac{\hat{V}_2}{8} = 0$

KNOW

$\hat{V}_2 - \hat{V}_1 = 5 \angle 0^\circ$

$\hat{I}_x = -\frac{\hat{V}_1}{-j2} = \frac{\hat{V}_1}{j2}$

add N1 & N2 and substitute for \hat{I}_x

$\hat{V}_1 (0.59 \angle 57.99^\circ) + \hat{V}_2 (.18 \angle 45^\circ) = 2 \angle 30^\circ$

$-\hat{V}_1 + \hat{V}_2 = 5 \angle 0^\circ$

~~solve~~ solve: $\hat{V}_1 = 2.00 \angle -58.65^\circ \text{ V}$

$\hat{V}_2 = 6.28 \angle -15.796^\circ \text{ V}$

$\hat{I}_x = 1.00 \angle -148.64^\circ \text{ A}$

$\hat{I}_y = 1.75 \angle 60.3^\circ \text{ A}$

$\hat{V}_1 = 21.78 \angle -32.52^\circ \text{ V}$

Problem Set 1

6. continued

mesh (see original circuit for mesh labels)

$$m1: \hat{V}_1 - 10\hat{I}_1 - (-j2)(\hat{I}_1 - \hat{I}_2) = 0$$

$$m2: -(-j2)(\hat{I}_2 - \hat{I}_1) + 5\angle 0^\circ - j8(\hat{I}_2 - \hat{I}_3) - 5\hat{I}_x = 0$$

$$m3: 5\hat{I}_x - j8(\hat{I}_3 - \hat{I}_2) - 8\hat{I}_3 = 0$$

$$\text{Known: } \hat{I}_x = \hat{I}_2 - \hat{I}_1 \quad \begin{matrix} \hat{I}_1 = 2\angle -30^\circ \text{ A} \\ \hat{I}_y = \hat{I}_2 \end{matrix}$$

simplify

$$m1: \hat{I}_2(-j2) + \hat{V}_1 = 20.40\angle -41.31^\circ$$

$$m2: \hat{I}_2(-5-j6) + \hat{I}_3(j8) = 14.41\angle 144.02^\circ$$

$$m3: \hat{I}_2(5+j8) + \hat{I}_3(-8-j8) = 10\angle -30^\circ$$

$$\text{solving } m2 \text{ \& } m3: \hat{I}_2 = 1.74\angle -60.47^\circ \text{ A}$$

$$\hat{I}_3 = 0.78\angle -14.13^\circ \text{ A}$$

$$\hat{I}_x = 1.01\angle -149.55^\circ \text{ A}$$

$$\hat{I}_y = 1.74\angle -60.47^\circ \text{ A}$$

$$\hat{V}_1 = 21.79\angle -32.63^\circ \text{ V}$$

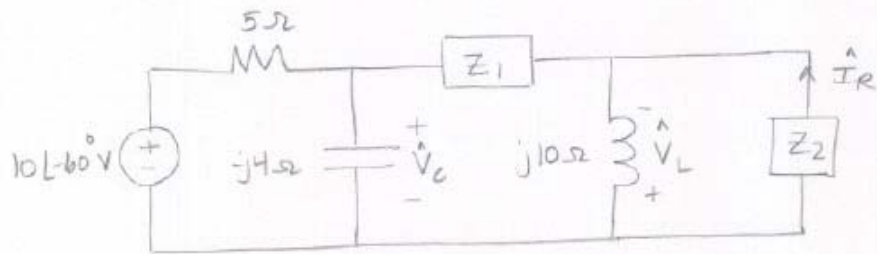
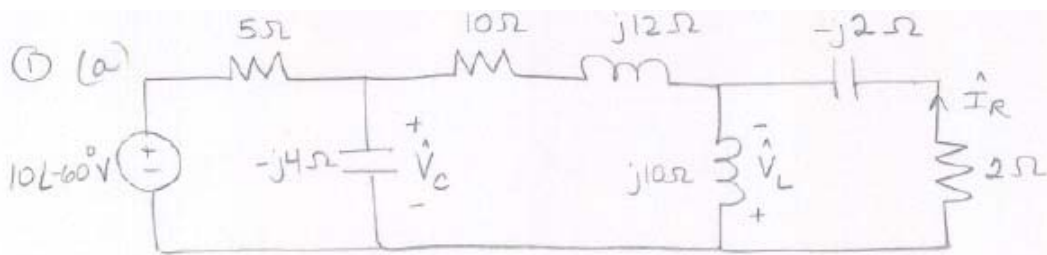
$$i_x(t) = 1.01 \cos(2t - 149.55^\circ) \text{ A}$$

$$i_y(t) = 1.74 \cos(2t - 60.47^\circ) \text{ A}$$

$$v_1(t) = 21.79 \cos(2t - 32.63^\circ) \text{ V}$$

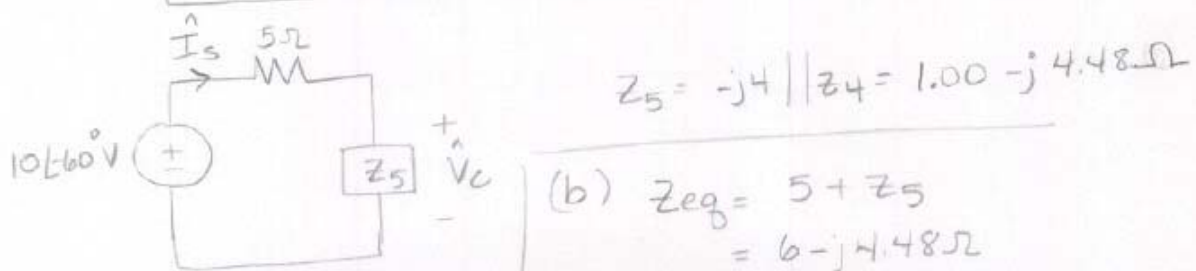
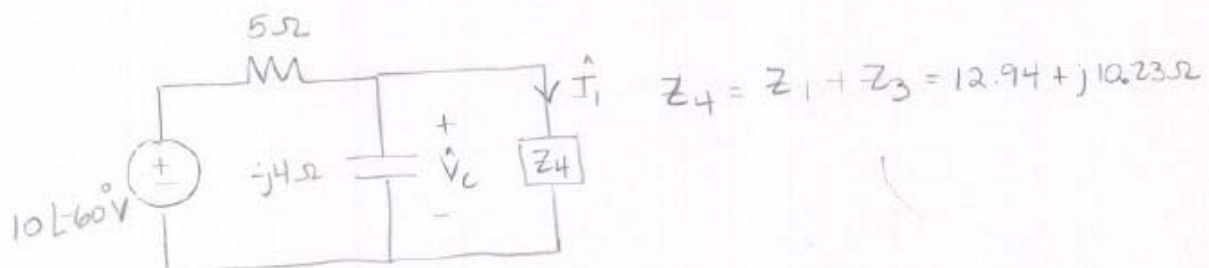
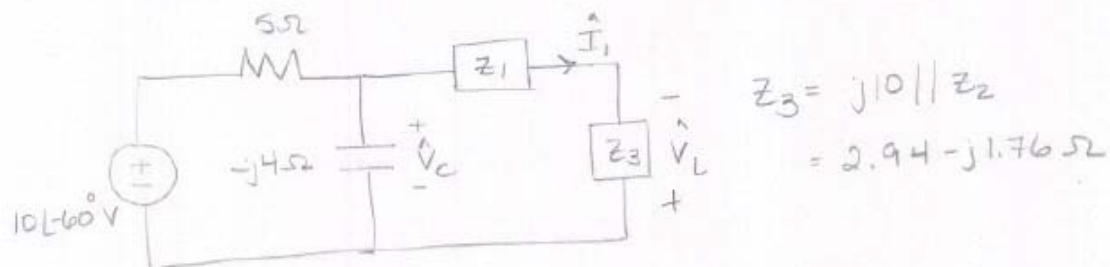
Problem Set 1 – Solutions

7.



$$Z_1 = 10 + j12 \Omega$$

$$Z_2 = 2 - j2 \Omega$$



$$(b) \quad Z_{eq} = 5 + Z_5$$

$$= 6 - j4.48 \Omega$$

$$X_{eq} = -4.48 \Omega \Rightarrow \text{capacitive}$$

Problem Set 1 - Solutions
7. continued

$$\textcircled{1} \quad \hat{I}_S = 1.33 \angle -23.25^\circ \text{ A}$$
$$\hat{V}_C = 10 \angle -60^\circ \cdot \frac{Z_5}{Z_5 + 5} = 6.13 \angle -100.63^\circ \text{ V}$$

$$\hat{I}_1 = \frac{\hat{V}_C}{Z_4} = 0.37 \angle -138.96^\circ \text{ A}$$

$$\hat{V}_L = -Z_3 \hat{I}_1 = 1.27 \angle 10.01^\circ \text{ V}$$

$$\hat{I}_R = \frac{\hat{V}_L}{Z_2} = 0.45 \angle 55.09^\circ \text{ A}$$

$$i_R(t) = 0.45 \cos(t + 55.09^\circ) \text{ A}$$

$$v_L(t) = 1.27 \cos(t + 10.01^\circ) \text{ V}$$

$$v_C(t) = 6.13 \cos(t - 100.63^\circ) \text{ V}$$