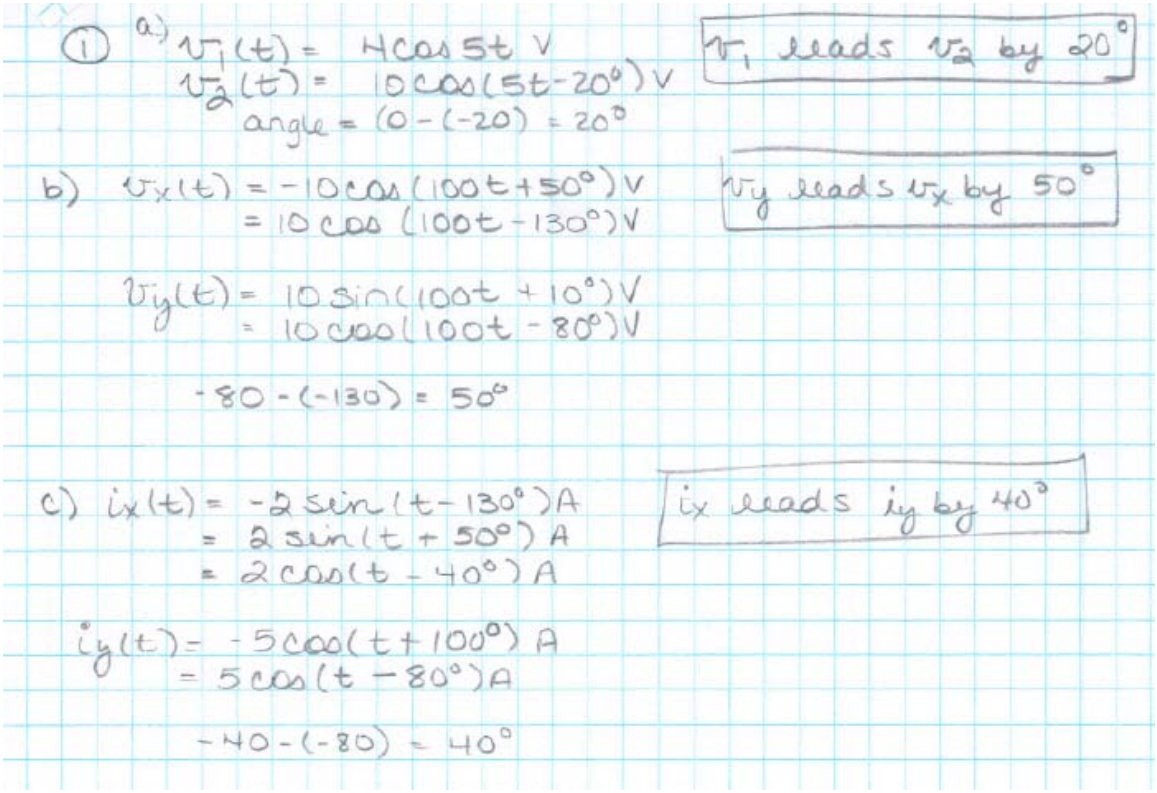
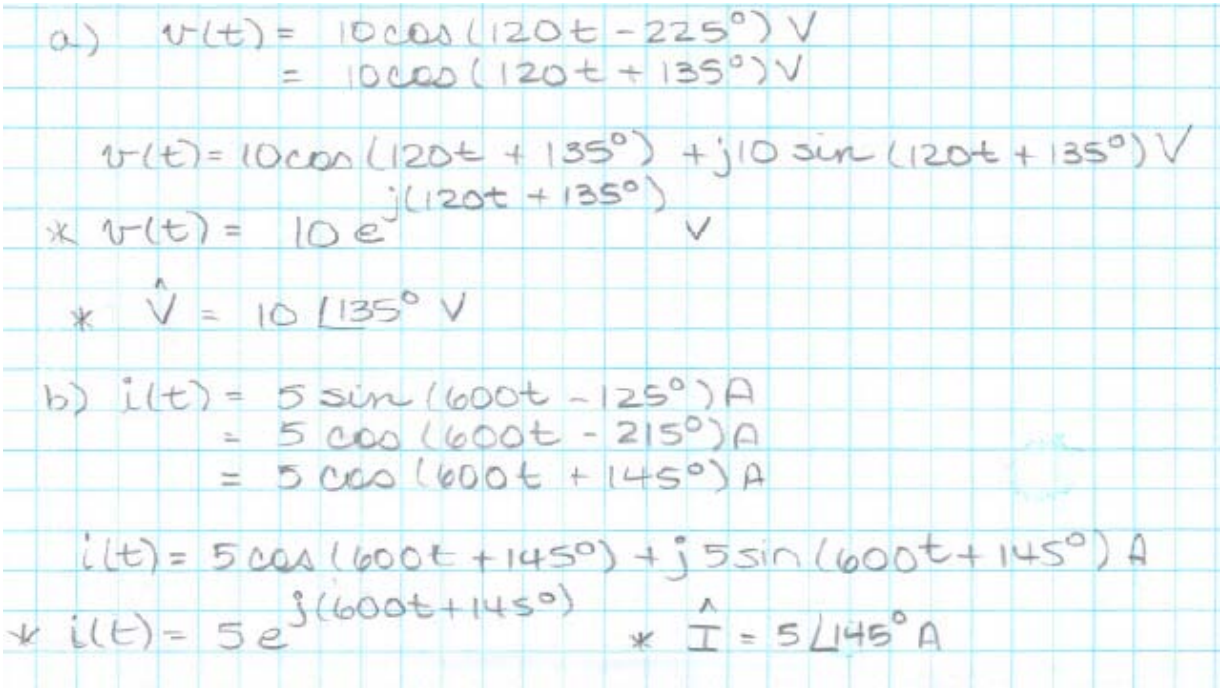
Problem Set 1 – Solutions

1.



2.



Problem Set 1 – Solutions

2. continued

c) $v(t) = -3\sin(20t)\ V$
$= 3\sin(20t + 180°)\ V$
$= 3\cos(20t + 90°)\ V$

$v(t) = 3\cos(20t + 90°) + j3\sin(20t + 90°)\ V$
$= 3e^{j(20t+90°)}\ V$
$\hat{V} = 3\angle 90°\ V$

d) $i(t) = -10\cos(2t + 45°)\ A$
$= 10\cos(2t - 135°)\ A$

$i(t) = 10\cos(2t - 135°) + j10\sin(2t + 135°)\ A$
$i(t) = 10e^{j(2t-135°)}$
$\hat{I} = 10\angle -135\ A$

3.

(3) $\omega = 5\ rad/s$

a) $6\angle 25 + 10\angle -40$

$6\angle 25 = 6\cos 25 + j6\sin 25$
$= 5.44 + j2.54$

$(10\angle -40) = 10\cos(-40) + j10\sin(-40)$
$= 7.66 - j6.43$

$(5.44 + j2.54)$
$+ (7.66 - j6.43)$
$= 13.10 - j3.89$

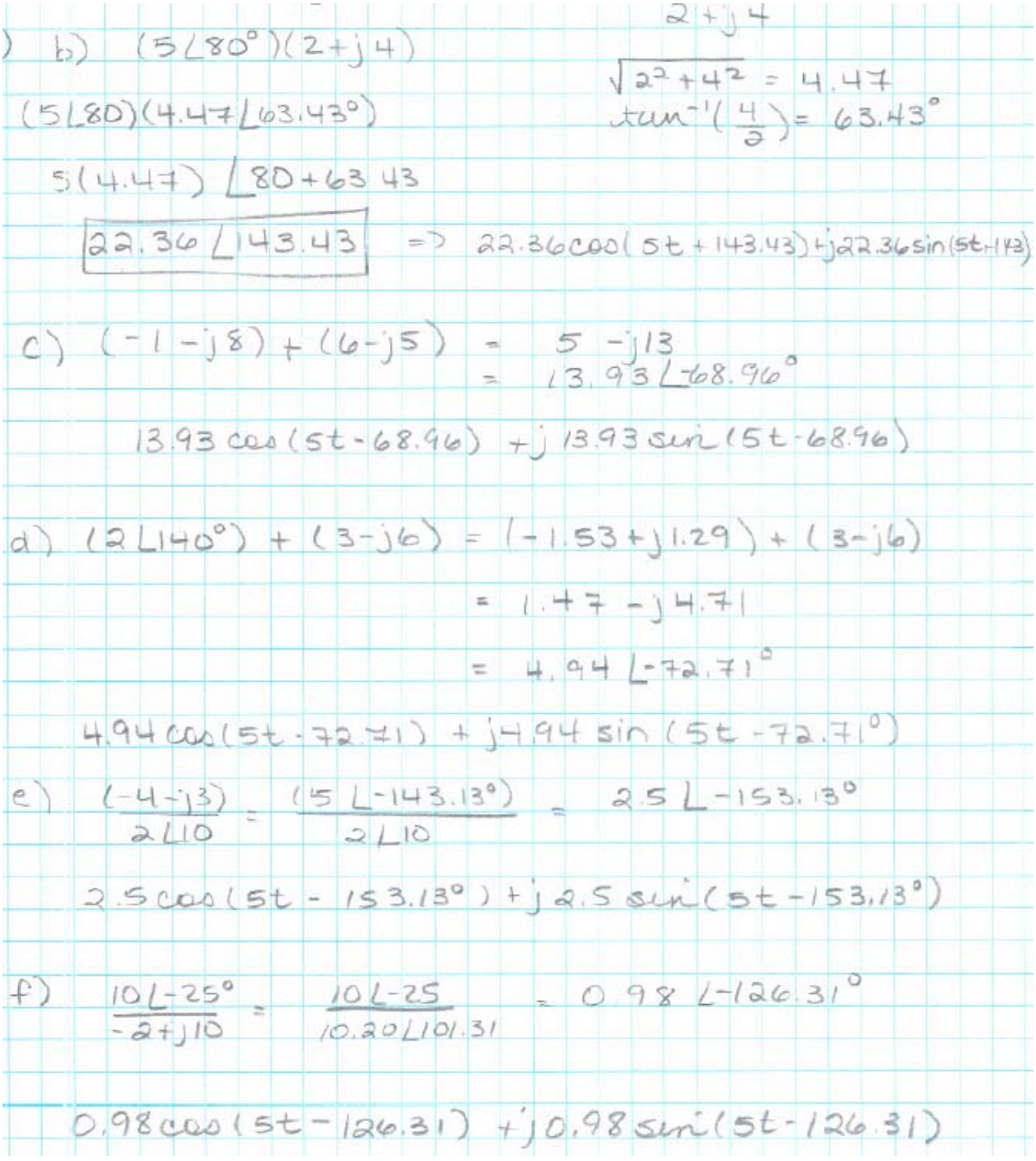
$\sqrt{(13.10)^2 + (3.89)^2} = 13.66$

$\tan^{-1}\left(\frac{-3.89}{13.10}\right) = -16.55°$

$13.66\angle -16.55° \Rightarrow 13.66\cos(5t - 16.55°) + j13.66\sin(5t - 16.55)$

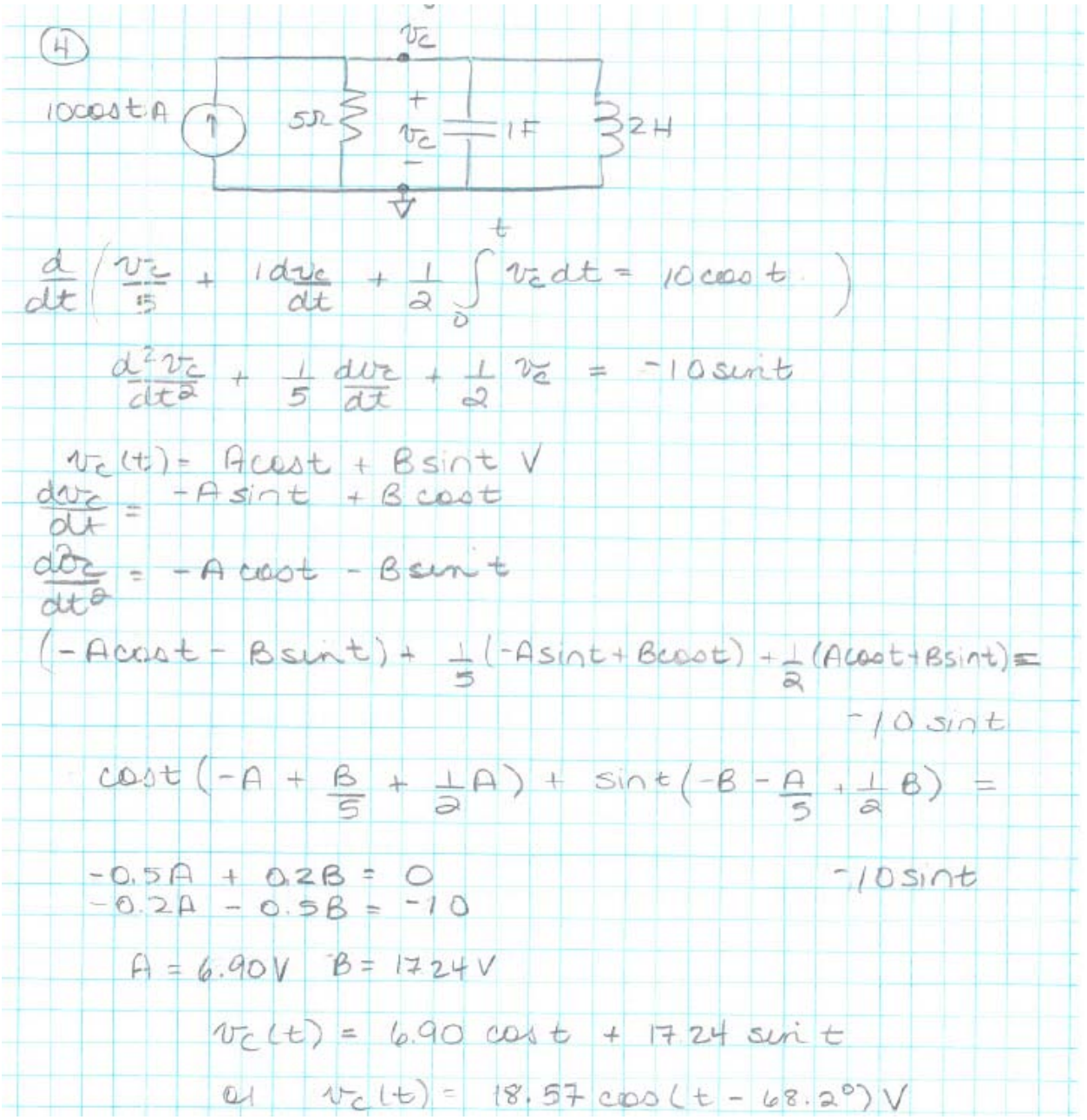
Problem Set 1 – Solutions

3. continued



Problem Set 1 – Solutions

4.



$$\frac{d}{dt}\left(\frac{v_c}{5} + 1\frac{dv_c}{dt} + \frac{1}{2}\int_0^t v_c\,dt = 10\cos t\right)$$

$$\frac{d^2v_c}{dt^2} + \frac{1}{5}\frac{dv_c}{dt} + \frac{1}{2}v_c = -10\sin t$$

$v_c(t) = A\cos t + B\sin t$ V

$\frac{dv_c}{dt} = -A\sin t + B\cos t$

$\frac{d^2v_c}{dt^2} = -A\cos t - B\sin t$

$(-A\cos t - B\sin t) + \frac{1}{5}(-A\sin t + B\cos t) + \frac{1}{2}(A\cos t + B\sin t) = -10\sin t$

$\cos t\left(-A + \frac{B}{5} + \frac{1}{2}A\right) + \sin t\left(-B - \frac{A}{5} + \frac{1}{2}B\right) = -10\sin t$

$-0.5A + 0.2B = 0$

$-0.2A - 0.5B = -10$

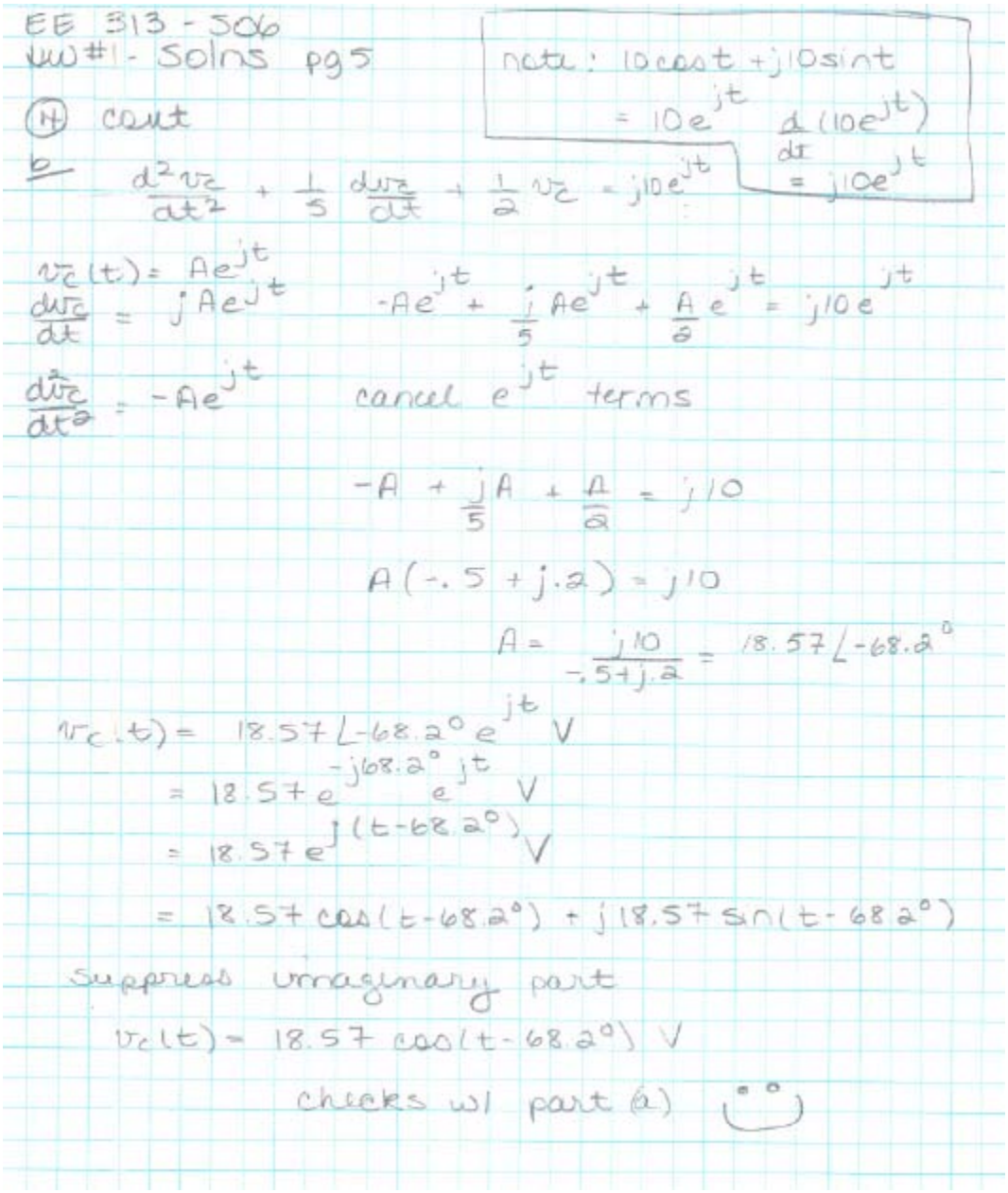
$A = 6.90$ V $B = 17.24$ V

$v_c(t) = 6.90\cos t + 17.24\sin t$

$v_c(t) = 18.57\cos(t - 68.2°)$ V

Problem Set 1 –Solutions

4. continued



4. continued

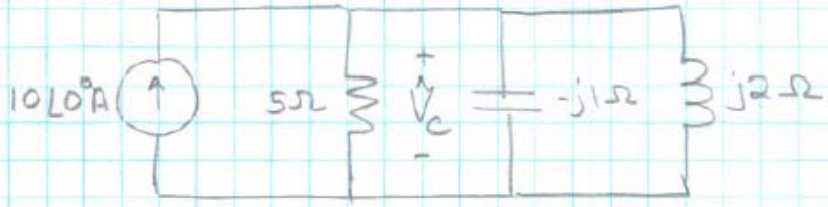
(d) check using Frequency Domain Analysis

$10\cos t\ A \rightarrow 10\angle 0^\circ\ A \qquad \omega = 1\ rad/s$

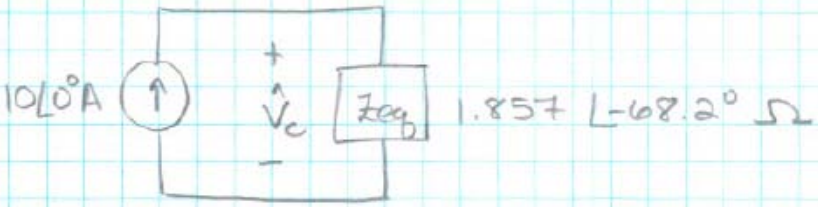
$5\Omega \rightarrow 5\Omega$

$1\ F \rightarrow -j1\Omega$

$2\ H \rightarrow j2\Omega$



$Z_{eq} = 5\Omega \,||\, -j1 \,||\, j2 = 1.857\angle -68.2^\circ\ \Omega$



$\hat{V}_c = (10\angle 0)\, Z_{eq} = 18.57\angle -68.2^\circ\ V$

so $v_c(t) = 18.57\cos(t - 68.2^\circ)\ V$

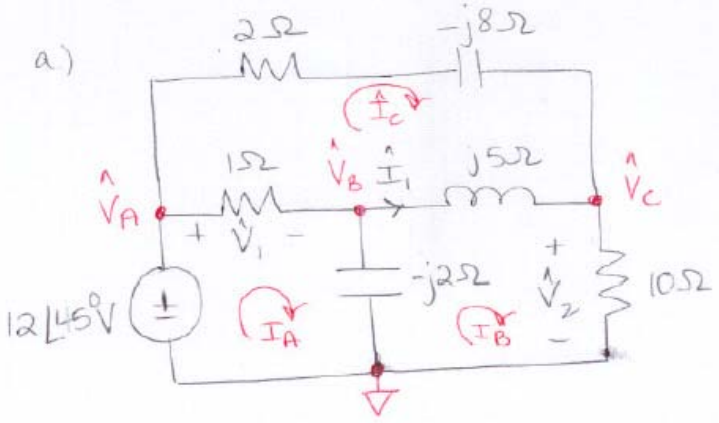
Checks w/ parts a + b

Problem Set 1 - Solutions

5.

pg 1

① a)



Know

$\hat{V}_A = 12\angle 45^\circ V$

b)

Node A: don't need

Node B: $\frac{\hat{V}_B - \hat{V}_A}{1} + \frac{\hat{V}_B}{-j2} + \frac{\hat{V}_B - \hat{V}_C}{j5} = 0$

Node C: $\frac{\hat{V}_C - \hat{V}_B}{j5} + \frac{\hat{V}_C - \hat{V}_A}{2-j8} + \frac{\hat{V}_C}{10} = 0$

$\hat{V}_2 = \hat{V}_C \qquad \hat{V}_1 = \hat{V}_A - \hat{V}_B \qquad \hat{I}_1 = \frac{\hat{V}_B - \hat{V}_C}{j5\Omega}$

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simplify

Node B: $\hat{V}_B(1.04\angle 16.7^\circ) + \hat{V}_C(.2\angle 90^\circ) = 12\angle 45^\circ$

Node C: $\hat{V}_B(.2\angle 90) + \hat{V}_C(0.15\angle -32.47^\circ) = 1.46\angle 120.96^\circ$

solve $\hat{V}_B = 10.67\angle 27.65^\circ\ V$

$\hat{V}_C = 4.55\angle -36.99^\circ\ V$

$\hat{V}_1 = 3.66\angle 105.29^\circ\ V$

$\hat{V}_2 = 4.55\angle -36.99^\circ\ V$

$\hat{I}_1 = 1.93\angle -37.11^\circ\ A$

Problem Set 1 – Solutions

5. continued

mesh (see original circuit for mesh labels)

mesh A: $12\angle 45 - 1(\hat{I}_A - \hat{I}_C) - (-j2)(\hat{I}_A - \hat{I}_B) = 0$

mesh B: $-(-j2)(\hat{I}_B - \hat{I}_A) - j5(\hat{I}_B - \hat{I}_C) - 10\hat{I}_B = 0$

mesh C: $-(2-j8)\hat{I}_C - j5(\hat{I}_C - \hat{I}_B) - 1(\hat{I}_C - \hat{I}_A) = 0$

$\hat{I}_1 = \hat{I}_B - \hat{I}_C \qquad \hat{V}_1 = 1(\hat{I}_A - \hat{I}_C) \qquad \hat{V}_2 = 10\hat{I}_B$

---

simplify

mesh A: $\hat{I}_A(-1+j2) + \hat{I}_B(-j2) + \hat{I}_C(1) = 12\angle -135$

mesh B: $\hat{I}_A(-j2) + \hat{I}_B(-10-j3) + \hat{I}_C(j5) = 0$

mesh C: $\hat{I}_A(1) + \hat{I}_B(j5) + \hat{I}_C(-3+j3) = 0$

$\hat{I}_A = 4.93\angle 115.44^\circ$ A

$\hat{I}_B = 0.445\angle -36.94^\circ$ A

$\hat{I}_C = 1.48\angle +142.13^{\circ\circ}$ A

$\hat{I}_1 = 1.93\angle -37.66^\circ$ A

$\hat{V}_1 = 4.45\angle -36.94^\circ$ V

$\hat{V}_2 = 3.67\angle 105^\circ$ V

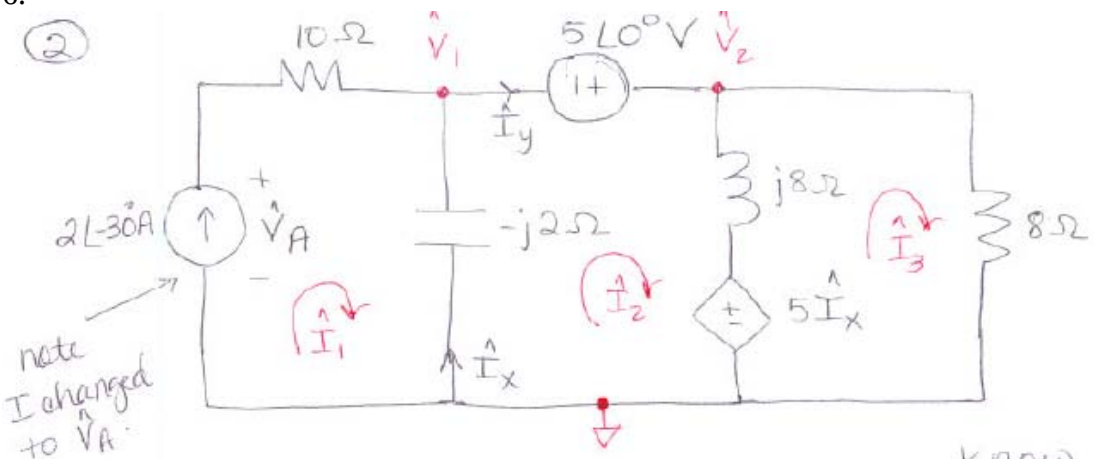
$i_1(t) = 1.93\cos(t - 37.66)$ A

$v_1(t) = 4.45\cos(t - 36.94^\circ)$ V

$v_2(t) = 3.67\cos(t + 105^\circ)$ V

Problem Set 1 – Solutions

6.



Node 1 : $2\angle 150 + \frac{\hat{V}_1}{-j2} + \hat{I}_y = 0$

Node 2 : $-\hat{I}_y + \frac{\hat{V}_2 - 5\hat{I}_x}{j8\Omega} + \frac{\hat{V}_2}{8} = 0$

know

$\hat{V}_2 - \hat{V}_1 = 5\angle 0$

$\hat{I}_x = -\frac{\hat{V}_1}{-j2} = \frac{\hat{V}_1}{j2}$

add N1 & N2 and substitute for $\hat{I}_x$

$\hat{V}_1(0.59\angle 57.99) + \hat{V}_2(.18\angle 45) = 2\angle -30$

$-\hat{V}_1 + \hat{V}_2 = 5\angle 0$

solve : $\hat{V}_1 = 2.00\angle -58.65^\circ$ V

$\hat{V}_2 = 6.28\angle -15.796^\circ$ V

$\hat{I}_x = 1.00\angle -148.64$ A

$\hat{I}_y = 1.75\angle -60.3^\circ$ A

$\hat{V}_1 = 21.78\angle -32.52$ V

Problem Set 1

6. continued

mesh (see original circuit for mesh labels)

m1: $\hat{V}_1 - 10\hat{I}_1 - (-j2)(\hat{I}_1 - \hat{I}_2) = 0$

m2: $-(-j2)(\hat{I}_2 - \hat{I}_1) + 5\angle 0^\circ - j8(\hat{I}_2 - \hat{I}_3) - 5\hat{I}_x = 0$

m3: $5\hat{I}_x - j8(\hat{I}_3 - \hat{I}_2) - 8\hat{I}_3 = 0$

Known: $\hat{I}_x = \hat{I}_2 - \hat{I}_1$ $\quad \hat{I}_1 = 2\angle -30^\circ A$

$\hat{I}_y = \hat{I}_2$

simplify

m1: $\hat{I}_2(-j2) + \hat{V}_1 = 20.40\angle -41.31^\circ$

m2: $\hat{I}_2(-5 - j6) + \hat{I}_3(j8) = 14.41\angle 144.02^\circ$

m3: $\hat{I}_2(5 + j8) + \hat{I}_3(-8 - j8) = 10\angle -30^\circ$

solving m2 & m3: $\hat{I}_2 = 1.74\angle -60.47^\circ A$

$\hat{I}_3 = 0.78\angle -16.13^\circ A$

$\hat{I}_x = 1.01\angle -149.55^\circ A$

$\hat{I}_y = 1.74\angle -60.47^\circ A$

$\hat{V}_1 = 21.79\angle -32.63^\circ V$

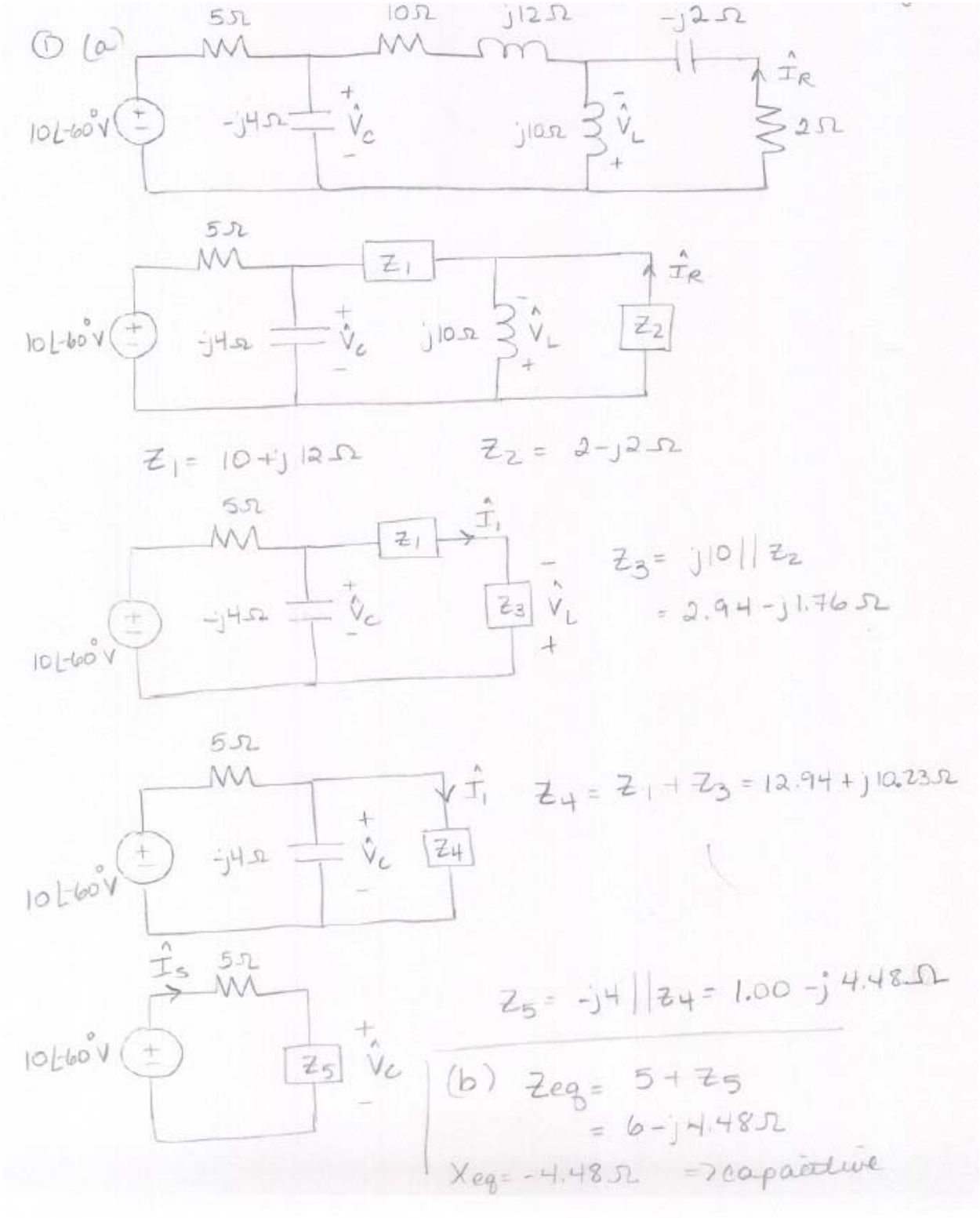
$i_x(t) = 1.01\cos(2t - 149.55^\circ)\ A$

$i_y(t) = 1.74\cos(2t - 60.47^\circ)\ A$

$v_1(t) = 21.79\cos(2t - 32.63^\circ)\ V$

Problem Set 1 – Solutions

7.



Problem Set 1 - Solutions
7. continued