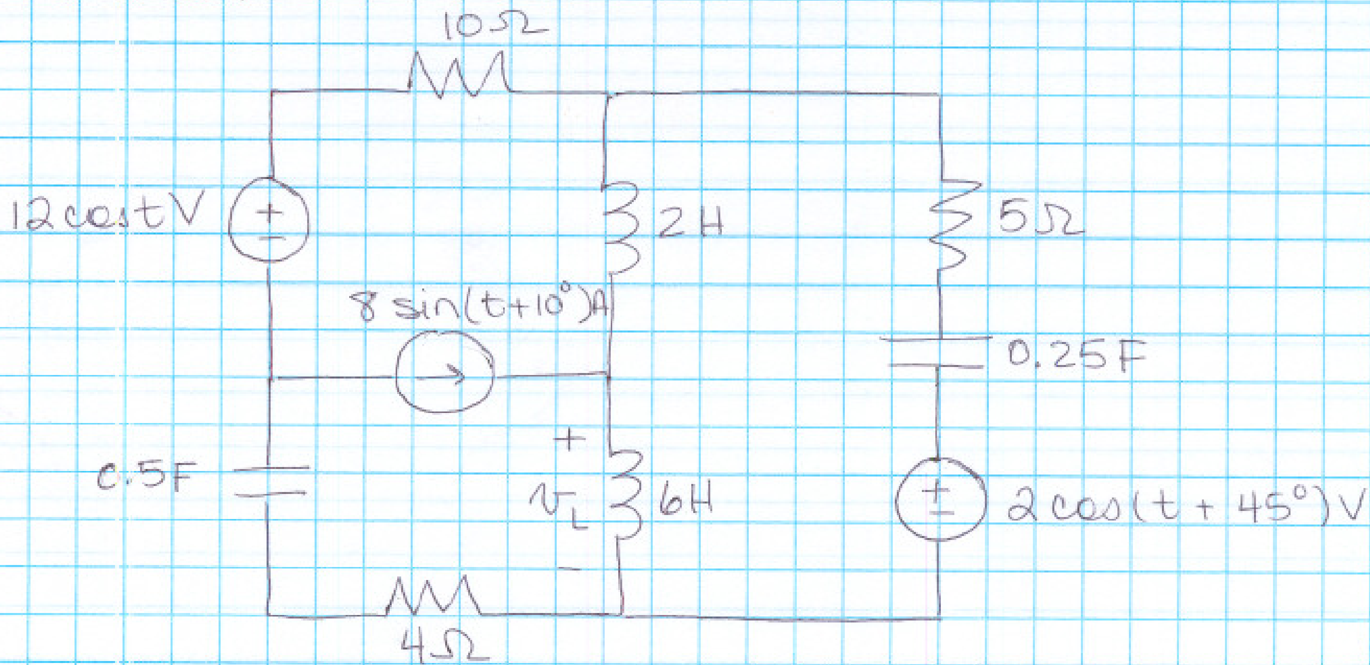
Practice Problems

Consider the circuit given in the time domain.



a) Convert the circuit to the frequency domain.

$\omega = 1$ rad/s

$12\cos t \rightarrow 12\angle 0°$ V

$2\cos(t+45°) \rightarrow 2\angle 45°$ V

$8\sin(t+10°) \rightarrow 8\angle -80°$ A

$10\Omega \rightarrow 10\Omega$

$2H \rightarrow j2\Omega$

$6H \rightarrow j6\Omega$

$0.5F \rightarrow -j2\Omega$

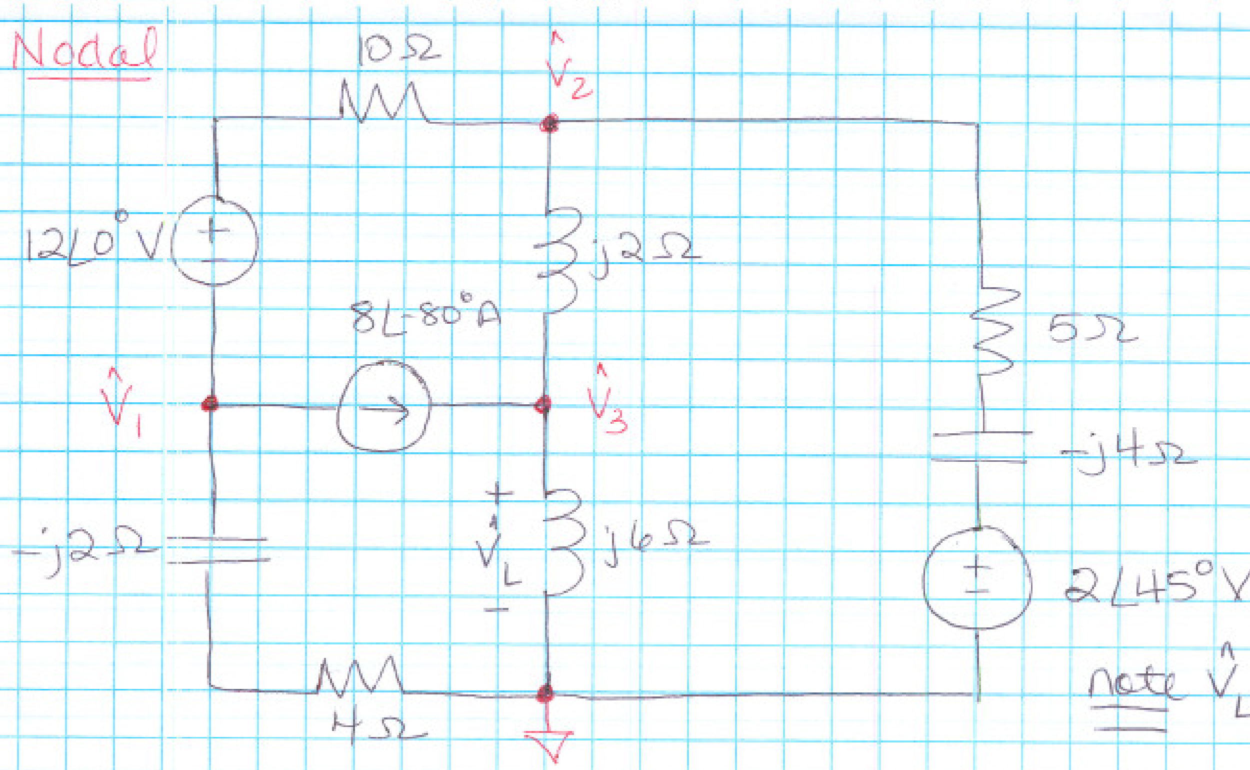
$4\Omega \rightarrow 2\Omega$

$5\Omega \rightarrow 5\Omega$

$0.25F \rightarrow -j4\Omega$

b) Solve for the real part of the time domain voltage using nodal analysis, mesh analysis, superposition and thevenin equivalent ckt.

Nodal



note $\hat{V}_L = \hat{V}_3$

N1: $\dfrac{\hat{V}_1 + 12\angle 0 - \hat{V}_2}{10} + \dfrac{\hat{V}_1}{4-j2} + 8\angle -80° = 0$

N2: $\dfrac{\hat{V}_2 - \hat{V}_1 - 12\angle 0}{10} + \dfrac{\hat{V}_2 - 2\angle 45}{5-j4} + \dfrac{\hat{V}_2 - \hat{V}_3}{j2} = 0$

N3: $\dfrac{\hat{V}_3 - \hat{V}_2}{j2} + \dfrac{\hat{V}_3}{j6} + 8\angle 100 = 0$

Simplify

N1 $\quad \hat{V}_1\left(\dfrac{1}{10} + \dfrac{1}{4-j2}\right) + \hat{V}_2\left(\dfrac{-1}{10}\right) = 1.2\angle 180 + 8\angle 100$

$\hat{V}_1\left(\dfrac{4-j2+10}{10(4-j2)}\right) + \hat{V}_2(-.1) = 8.29\angle 108.19$

$\hat{V}_1\left(\dfrac{14-j2}{40-j20}\right) + \hat{V}_2(-.1) = 8.29\angle 108.19$

$\boxed{\hat{V}_1(.316\angle 18.43) + \hat{V}_2(-.1) = 8.29\angle 108.19}$

<u>Nodal cont</u>

N2: $\hat{V}_1(-.1) + \hat{V}_2\left(\frac{1}{10} + \frac{1}{5-j4} + \frac{1}{j2}\right) + \hat{V}_3\left(-\frac{1}{j2}\right) = \frac{12\angle 0}{10} + \frac{2\angle 45}{5-j4}$

$$\hat{V}_1(-.1) + \hat{V}_2\left(\frac{(8+j10) + (j20) + (50-j40)}{10(5-j4)(j2)}\right) + \hat{V}_3(j0.5) = 1.27\angle 14.11^\circ$$

$$\hat{V}_1(-.1) + \hat{V}_2\left(\frac{58-j10}{80+j100}\right) + \hat{V}_3(j.5) = 1.27\angle 14.11^\circ$$

$$\boxed{\hat{V}_1(-.1) + \hat{V}_2(.46\angle -61.12) + \hat{V}_3(j.5) = 1.27\angle 14.11^\circ}$$

N3: $\hat{V}_2\left(-\frac{1}{j2}\right) + \hat{V}_3\left(\frac{1}{j2} + \frac{1}{j6}\right) = 8\angle -80^\circ$

$$\hat{V}_2(j.5) + \hat{V}_3\left(\frac{j8}{(j2)(j6)}\right) = 8\angle -80^\circ$$

$$\hat{V}_2(j.5) + \hat{V}_3\left(\frac{j8}{-12}\right) = 8\angle -80$$

$$\boxed{\hat{V}_2(j.5) + \hat{V}_3(.667\angle -90) = 8\angle -80^\circ}$$

3 equations / 3 unknowns note $j.5 = .5\angle 90^\circ$

$$\begin{bmatrix} .316\angle 18.43 & -.1 & 0 \\ -.1 & .46\angle -61.12 & j.5 \\ 0 & j.5 & .667\angle -90 \end{bmatrix} \begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \\ \hat{V}_3 \end{bmatrix} = \begin{bmatrix} 8.29\angle 108.19 \\ 1.27\angle 14.11 \\ 8\angle -80^\circ \end{bmatrix}$$

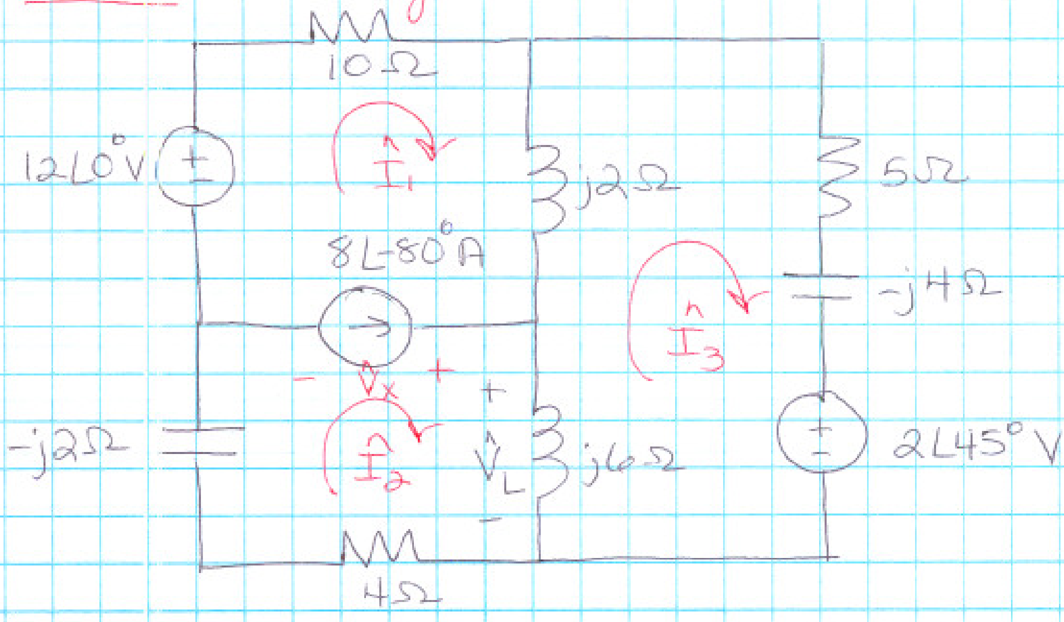
$\boxed{\hat{V}_1 = 20.81\angle 82.67^\circ V}$ $\boxed{\hat{V}_2 = 19.43\angle -47.12^\circ V}$ $\boxed{\hat{V}_3 = 23.36\angle -21.57^\circ V}$

Nodal cont

$$\hat{V}_L = \hat{V}_3 = 23.36\angle -21.57^\circ \text{ V}$$

$$\boxed{v_L(t) = 23.36\cos(t - 21.57^\circ)\text{ V}}$$

Mesh analysis



note
$\hat{V}_L = j6(\hat{I}_2 - \hat{I}_3)$
$\hat{I}_2 - \hat{I}_1 = 8\angle -80^\circ \text{ A}$
(I labeled $\hat{V}_x$

m1: $12\angle 0 - 10\hat{I}_1 - j2(\hat{I}_1 - \hat{I}_3) - \hat{V}_x = 0$

m2: $-(j2)(\hat{I}_2) + \hat{V}_x - j6(\hat{I}_2 - \hat{I}_3) - 4\hat{I}_2 = 0$

m3: $-j2(\hat{I}_3 - \hat{I}_1) - (5 - j4)\hat{I}_3 - (2\angle 45) - j6(\hat{I}_3 - \hat{I}_2) = 0$

Simplify

m1: $\hat{I}_1(-10 - j2) + \hat{I}_3(j2) - \hat{V}_x = 12\angle 180^\circ$

m2: $\hat{I}_2(-4 - j4) + \hat{I}_3(j6) + \hat{V}_x = 0$

m3: $\hat{I}_1(j2) + \hat{I}_2(j6) + \hat{I}_3(-5 - j4) = 2\angle 45$

Mesh Analysis

solve: w/ the 3 mesh equations and the known equation about the current source, there are 4 equations and 4 unknowns

However; we don't need $\hat{V}_X$ so I will add m1 and m2 together.

(m1+m2) $\hat{I}_1(-10-j2) + \hat{I}_2(4-j4) + \hat{I}_3(j8) = 12\angle 180^\circ$

(m3) $\hat{I}_1(j2) + \hat{I}_2(j6) + \hat{I}_3(-5-j4) = 2\angle 45^\circ$

Known $\hat{I}_1(-1) + \hat{I}_2(1) = 8\angle -80^\circ$.

$$\begin{bmatrix} -10-j2 & -4-j4 & j8 \\ j2 & j6 & -5-j4 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \\ \hat{I}_3 \end{bmatrix} = \begin{bmatrix} 12\angle 180^\circ \\ 2\angle 45^\circ \\ 8\angle -80^\circ \end{bmatrix}$$

$\hat{I}_1 = 3.49\angle 87.66^\circ$ A

$\hat{I}_2 = 4.65\angle -70.75^\circ$ A

$\hat{I}_3 = 3.07\angle -14.43^\circ$ A

$\hat{V}_L = j6(\hat{I}_2 - \hat{I}_3)$

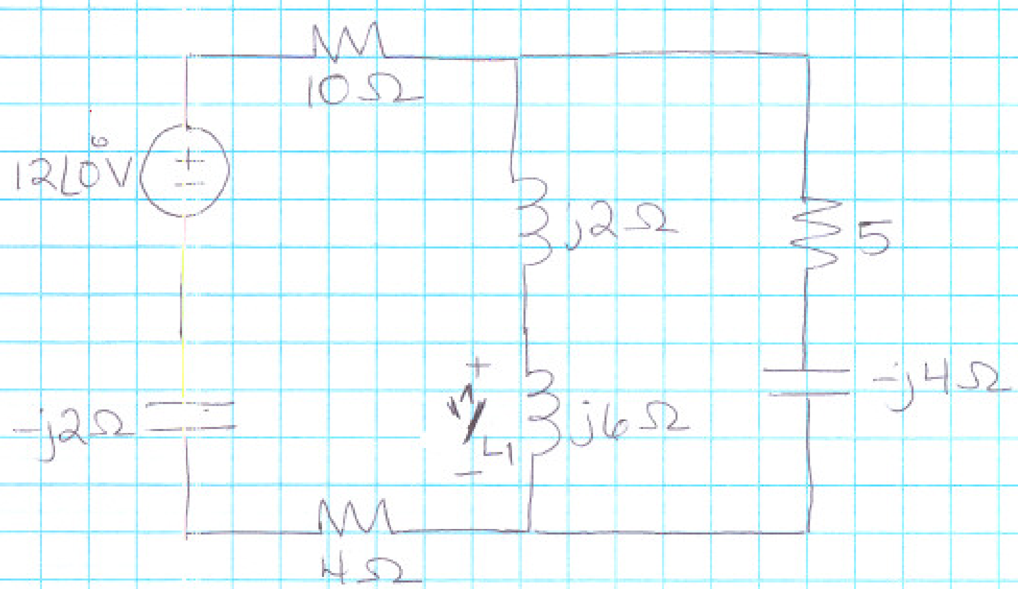
$= 23.40\angle -21.67^\circ$ V

$v_L(t) = 23.40\cos(t - 21.67^\circ)$ V

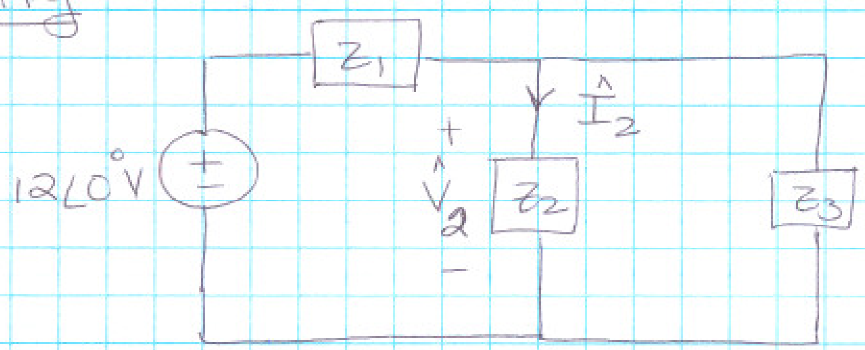
checks w/ modal :)

Superposition

$12\angle 0^\circ$ ON



simplify



note $\hat{V}_{L_1} = \hat{I}_2(j6)$

$Z_1 = 14 - j2\,\Omega$

$Z_2 = j8\,\Omega$

$Z_3 = 5 - j4\,\Omega$

using voltage division to solve for $\hat{V}_2$

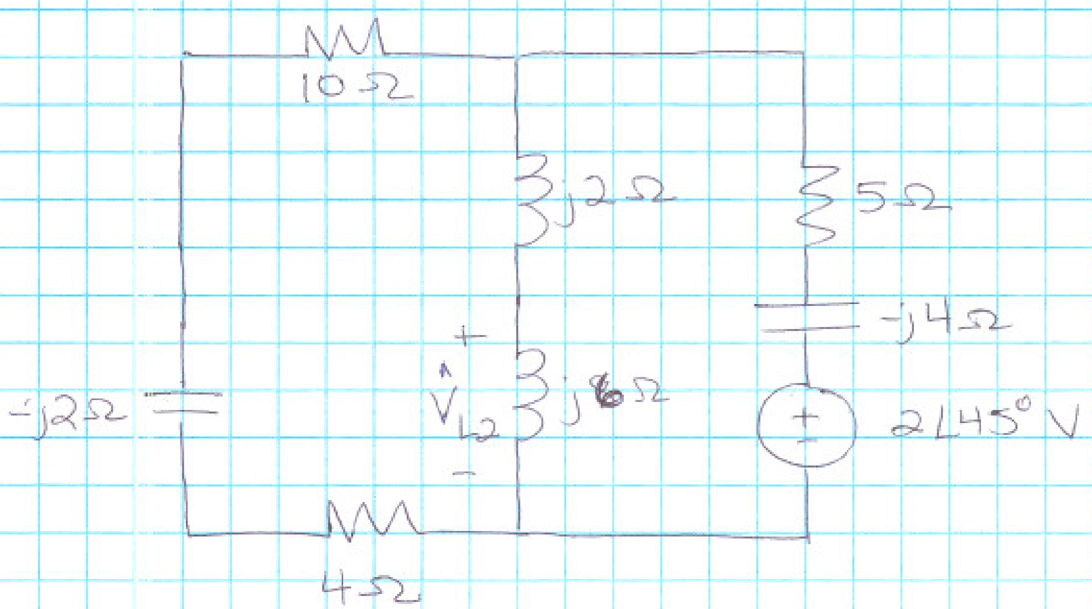
$$\hat{V}_2 = 12\angle 0 \cdot \frac{Z_2 \parallel Z_3}{Z_1 + (Z_2 \parallel Z_3)} = 4.40\angle 13.32^\circ\ \mathrm{V}$$

$$\hat{I}_2 = \frac{\hat{V}_2}{Z_2} = 0.55\angle -76.68^\circ\ \mathrm{A}$$

$$\boxed{\hat{V}_{L_1} = \hat{I}_2(j6) = 3.30\angle 13.32^\circ\ \mathrm{V}}$$

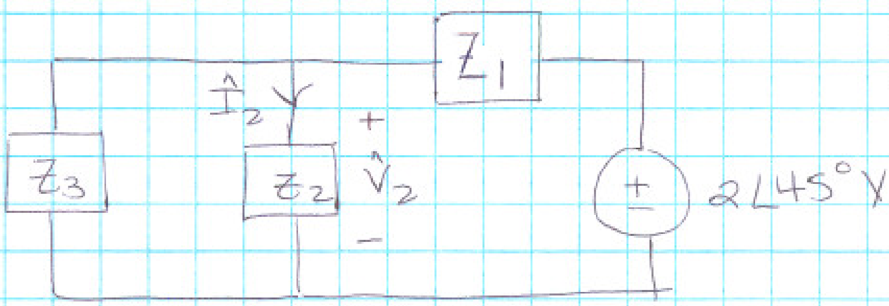
Superposition

$2\angle 45^\circ$ V ON



simplify

$Z_1 = 5 - j4\,\Omega$
$Z_2 = j8\,\Omega$
$Z_3 = 14 - j2\,\Omega$



again: $\hat{V}_{L2} = \hat{I}_2(j6)$

by voltage division $\quad \hat{V}_2 = 2\angle 45 \cdot \dfrac{Z_2 \| Z_3}{Z_1 + (Z_2 \| Z_3)}$

$\hat{V}_2 = 1.62\angle 88.85^\circ$ V

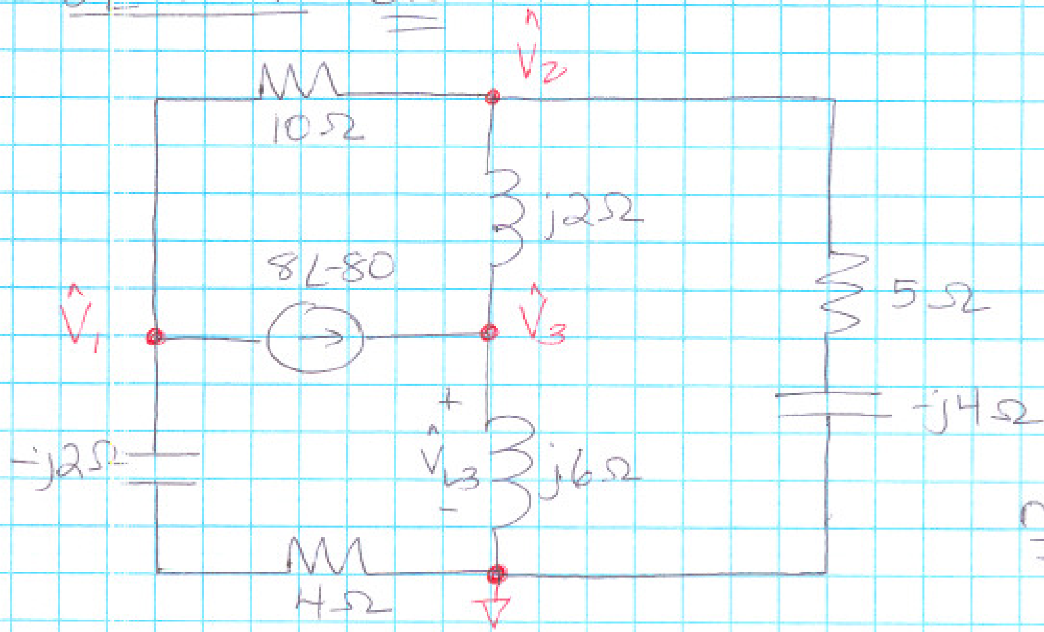
$\hat{I}_2 = \dfrac{\hat{V}_2}{Z_2} = 0.202\angle -1.15^\circ$ A

$\hat{V}_{L2} = \hat{I}_2(j6)$

$\boxed{\hat{V}_{L2} = 1.21\angle 88.85^\circ \text{ V}}$

Superposition

$8\angle -80^\circ A$ ON



Simplification is not a good option. I'm going to use nodal analysis

note $\hat{V}_{L3} = \hat{V}_3$

N1 $\frac{\hat{V}_1 - \hat{V}_2}{10} + \frac{\hat{V}_1}{4-j2} + 8\angle -80 = 0$ $\quad \hat{V}_1(.316\angle 18.43) + \hat{V}_2(-.1) = 8\angle 100$

N2 $\frac{\hat{V}_2 - \hat{V}_1}{10} + \frac{\hat{V}_2 - \hat{V}_3}{j2} + \frac{\hat{V}_2}{5-j4} = 0$ $\quad \hat{V}_1(-.1) + \hat{V}_2(.46\angle -61.12) + \hat{V}_3(.5\angle 90) = 0$

N3 $\frac{\hat{V}_3 - \hat{V}_2}{j2} + \frac{\hat{V}_3}{j6} + 8\angle 100 = 0$ $\quad \hat{V}_2(.5\angle 90) + \hat{V}_3(.67\angle 90) = 8\angle -80$

solving
$$\begin{bmatrix} .316\angle 18.43 & -.1 & 0 \\ -.1 & .46\angle -61.12 & .5\angle 90 \\ 0 & .5\angle 90 & .67\angle -90 \end{bmatrix} \begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \\ \hat{V}_3 \end{bmatrix} = \begin{bmatrix} 8\angle 100 \\ 0 \\ 8\angle -80 \end{bmatrix}$$

$\hat{V}_1 = 19.72\angle 76.04^\circ V$
$\hat{V}_2 = 18.93\angle -61.52^\circ V$
$\hat{V}_3 = 21.19\angle -29.22^\circ V$

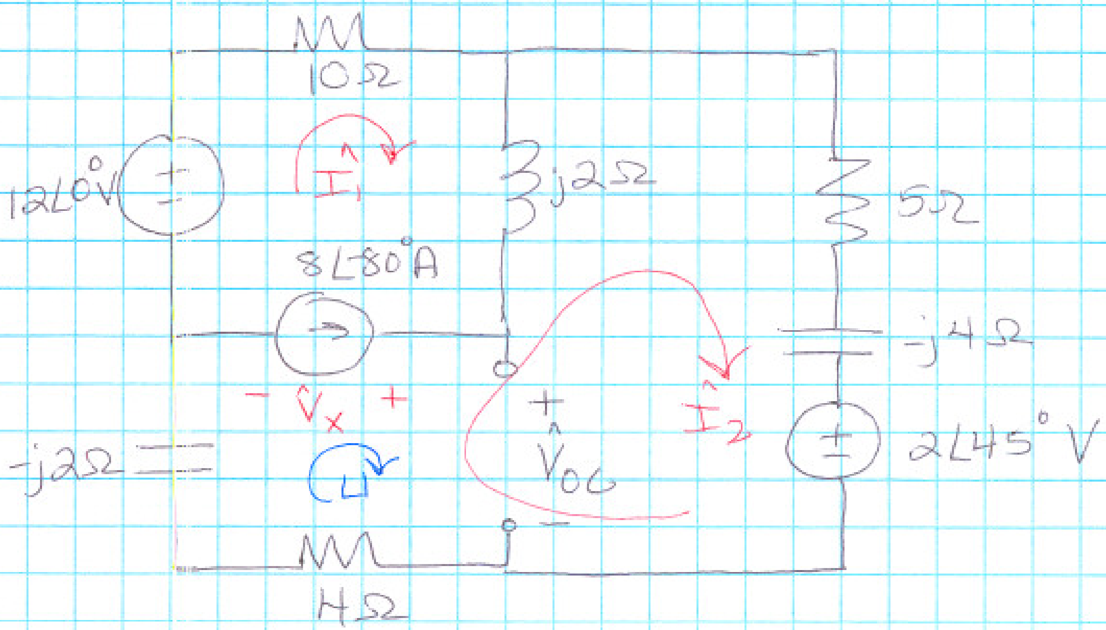
$\hat{V}_{L3} = 21.19\angle -29.22^\circ V$

$$\boxed{\begin{aligned} \hat{V}_L &= \hat{V}_{L1} + \hat{V}_{L2} + \hat{V}_{L3} \\ &= 23.29\angle -21.08^\circ V \\ v_L(t) &= 23.29\cos(t - 21.08^\circ) V \end{aligned}}$$

checks w/ nodal + mesh :)

Thevenin's

Ckt network (Load is $j6\Omega$)



note

I will use mesh analysis to solve

$\hat{I}_2 - \hat{I}_1 = 8\angle -80°$ (known)

m1 $\quad 12\angle 0 - 10\hat{I}_1 - j2(\hat{I}_1 - \hat{I}_2) - \hat{V}_x = 0$

m2 $\quad -(4-j2)\hat{I}_2 + \hat{V}_x - j2(\hat{I}_2 - \hat{I}_1) - (5-j4)\hat{I}_2 - 2\angle 45° = 0$

* write a KVL loop (see L1 in circuit) to find $\hat{V}_{oc}$

$$-(4-j2)\hat{I}_2 + \hat{V}_x - \hat{V}_{oc} = 0$$

using m1 & m2 and known

m1 $\quad \hat{I}_1(-10-j2) + \hat{I}_2(j2) - \hat{V}_x = 12\angle 180$

m2 $\quad \hat{I}_1(j2) + \hat{I}_2(-9+j4) + \hat{V}_x = 2\angle 45$

known $\quad \hat{I}_1(-1) + \hat{I}_2(1) = 8\angle -80$

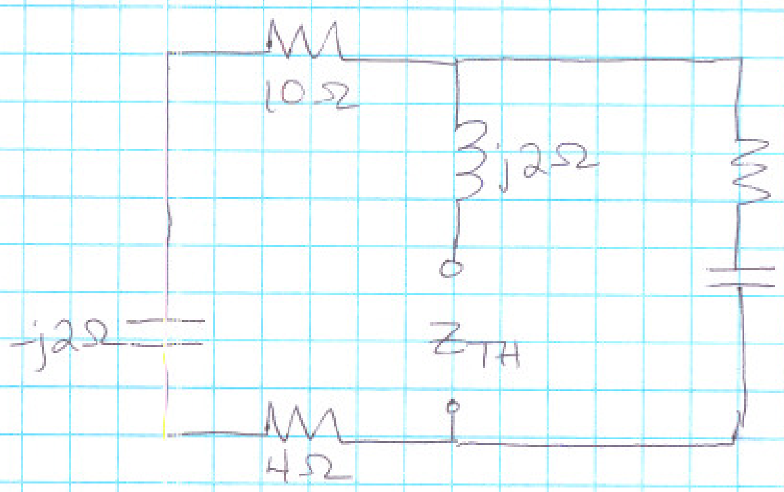
solve
$$\begin{bmatrix} -10-j2 & j2 & -1 \\ j2 & -9+j4 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \\ \hat{V}_x \end{bmatrix} = \begin{bmatrix} 12\angle 180 \\ 2\angle 45 \\ 8\angle -80 \end{bmatrix}$$

$\hat{I}_1 = 4.52\angle 77.29°$ A $\quad \hat{I}_2 = 4.21\angle -55.50°$ A $\quad \hat{V}_x = 45\angle -66.68°$ V solving for $\hat{V}_{OC}$

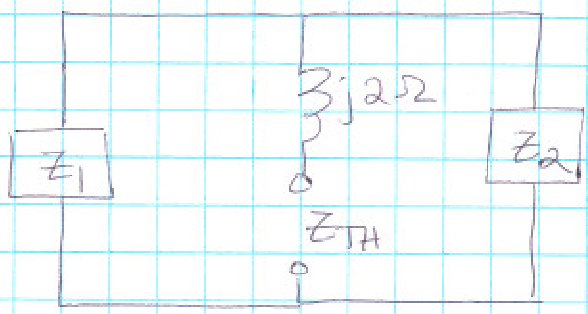
$$\hat{V}_{OC} = \hat{V}_X - (4-j2)\hat{I}_2 = 27.31\angle -56.14^\circ \text{ V}$$

$$\hat{V}_{TH} = \hat{V}_{OC}$$

Find Z_{TH} using Quick method.



Simplify



$Z_1 = 14 - j2\,\Omega$

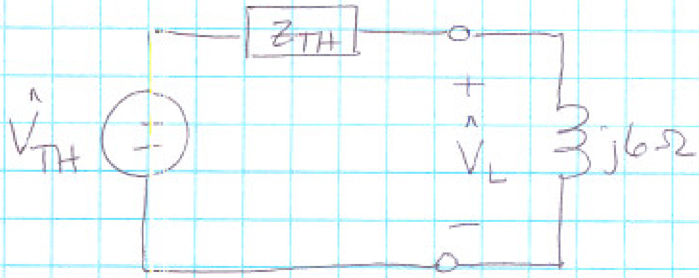
$Z_2 = 5 - j4\,\Omega$

$Z_{TH} = (Z_1 \| Z_2) + j2$

$= 3.97\angle -3.20^\circ\,\Omega$

or $3.96 - j0.22\,\Omega$

Build Thev. Equiv. ckt



$$\hat{V}_L = \hat{V}_{TH} \cdot \frac{j6}{Z_{TH} + j6}$$

$$= 23.38\angle -21.69^\circ \text{ V}$$

$$v_L(t) = 23.38\cos(t - 21.69^\circ) \text{ V}$$

Checks w/ other techniques