Preliminary Ph.D. Exam in Control Theory   Fall 2006

Show all your work (no credits for unsupported answer). Good luck!

1. Given a block-diagram of a closed loop control system ***(10 points)***

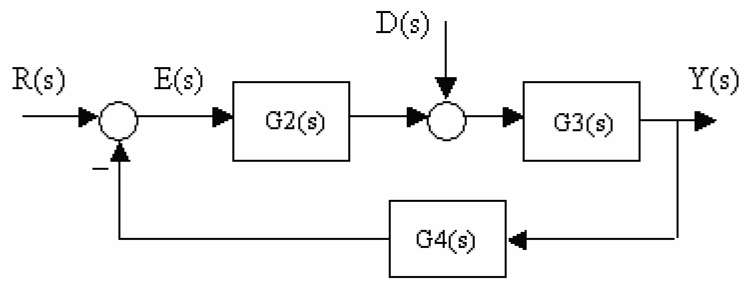


Fig. 1

- Assuming $D(s)=0$, $G2(s)=\frac{2}{s}$, $G4(s)=1.0$, and $G3(s)=\frac{3}{4s+1}$ identify the natural frequency and the damping ratio of the close-loop transfer function $W(s)=\frac{Y(s)}{R(s)}$. Is the close loop system under damped, over damped or critically damped?
- Given $r(t)$ and $d(t)$ the unit step functions predict the steady state error $e_{ss}=\lim_{t\to\infty} e(t)$.

2. Assuming (fig. 1) $R(s)=\frac{1}{s}$, $D(s)=0$, $G4(s)=1.0$, $G2(s)=K$, and ***(10 points)***

(a) $G3(s)=\frac{1}{s^3+as^2+bs+c}$ with $a=b=c=2$

(b) $G3(s)=\frac{1}{s^2+bs+c}$ with $b=c=2$

prove or prove wrong that "there does not exist value of $K>0$ that provides for an *arbitrary small* stabilization error, i.e. $\lim_{t\to\infty}|e(t)|=\varepsilon>0$, where $\varepsilon>0$ is arbitrary small" for (a) and (b).

3. Assuming (fig. 1) $D(s)=0$, $G4(s)=1$, $G2(s)=K>0$, and $G3(s)=\frac{50}{(s+1)^2(s+10)}$ identify a range for K that makes the close loop system stable using

- Nyquist criterion (give a geometric interpretation)
- Routh-Hurwitz criterion ***(10 points)***

4. Given a system described by a second order time invariant differential equation ***(10 points)***

$$\ddot{y}+3\dot{y}-5y=u+d(t)$$

Design a PID controller u that provides an asymptotic set point regulation, i.e. $\lim_{t\to\infty}|y_c(t)-y(t)|=0$ for $y_c(t)=const$ in presence of unknown constant disturbance $d(t)=const$.

5. Given a system described by a second order time invariant differential equation ***(10 points)***

$$\ddot{y} + 3\dot{y} - 5y = \alpha\dot{u} + u$$

(a) Assuming $\alpha = 0$ design control u that provides asymptotic *tracking* an arbitrary profile $y_c(t)$, i.e. $\lim_{t\to\infty}\left|y_c(t) - y(t)\right| = 0$ with the all eigenvalues of the characteristic equation of the compensated system to be equal to -3. How many times the arbitrary profile $y_c(t)$ must be differentiable?

(b) Assuming $\alpha = -3.5$ repeat the output *tracking* controller design. Does any problem in a tracking controller design associated with a negative sign of the coefficient α exist?

Preliminary Ph.D. Exam in Control Theory Spring 2006

Show all your work (no credits for unsupported answer). Good luck!

1. Given a system described by a second order time invariant differential equation ***(10 points)***

$$\ddot{y} + 5\dot{y} - 3y = \alpha\dot{u} + u$$

1.1 Assuming $\alpha = 0$ design control u that provides asymptotic tracking an arbitrary profile $y_c(t)$, i.e. $\lim_{t\to\infty}|y_c(t) - y(t)| = 0$ with the all eigenvalues of the characteristic equation of the compensated system to be equal to -3. How many times the arbitrary profile $y_c(t)$ must be differentiable?

1.2 Assuming $\alpha = -2$ repeat the controller design. Does any problem in a tracking controller design associated with a negative sign of the coefficient α exist?

2. Given a block-diagram of a closed loop control system ***(10 points)***

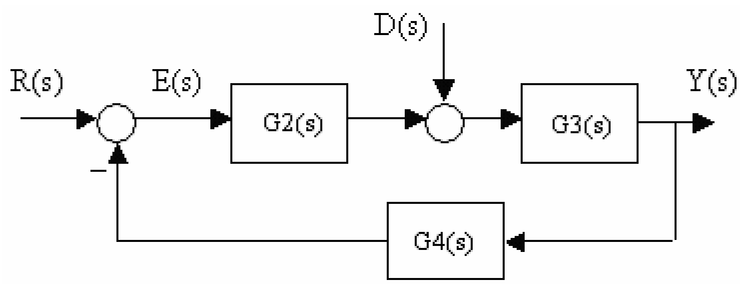


Fig. 1

2.1 Find the transfer functions $W_1(s)$ and $W_2(s)$ in the equation

$$Y(s) = W_1(s)R(s) + W_2(s)D(s)$$

2.2 Assuming $D(s) = 0$ and $G2(s) = \frac{1}{s}$, $G4(s) = 1.0$, and $G3(s) = \frac{4}{2s+1}$ identify the natural frequency and the damping ratio of the close-loop transfer function $W_1(s)$.

2.3 Given $r(t)$ and $d(t)$ a unit step function predict the steady state error.

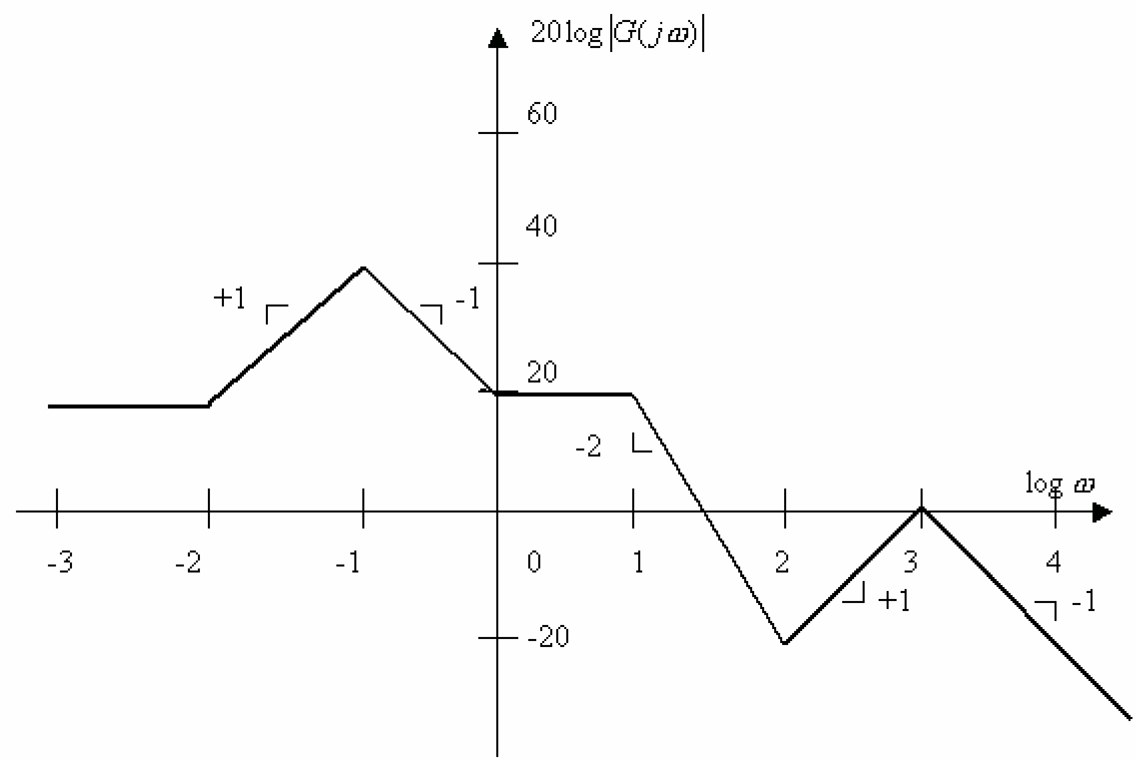
3. Assuming $G4(s) = 1$ and $G3(s) = \frac{5}{s(s-2)}$ (fig 1) design a dynamic compensator $G2(s)$ of minimal complexity to stabilize the closed loop control system. Make all the roots of the characteristic equation of the closed loop control system equal to -4. ***(10 points)***

4. Assuming $G4(s) = 1$, $G2(s) = K$, and $G3(s) = \frac{1}{s^3 + as^2 + bs + c}$ (fig. 1) with $a = b = c = 3$

4.1 prove or prove wrong that "there does not exist value of $K > 0$ that provides for an *arbitrary small* stabilization error, i.e. $\lim_{t\to\infty}|e(t)| = \varepsilon > 0$, where $\varepsilon > 0$ is arbitrary small";

4.2 using Routh-Hurwitz criterion and auxiliary polynomial technique, identify $K > 0$, which makes the close-loop system (fig. 1) marginally stable; identify a corresponding frequency of oscillations. ***(10 points)***

5. Given the asymptotic magnitude Bode plot of a minimum phase transfer function G(s) identify the transfer function G(s). ***(10 points)***



Preliminary Ph.D. Exam in Control Theory Spring 2008

Show all your work (no credits for unsupported answer). Good luck!

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Show all your work (no credits for unsupported answer). Close book. Good luck!

A block-diagram of an armature controlled DC motor is given.

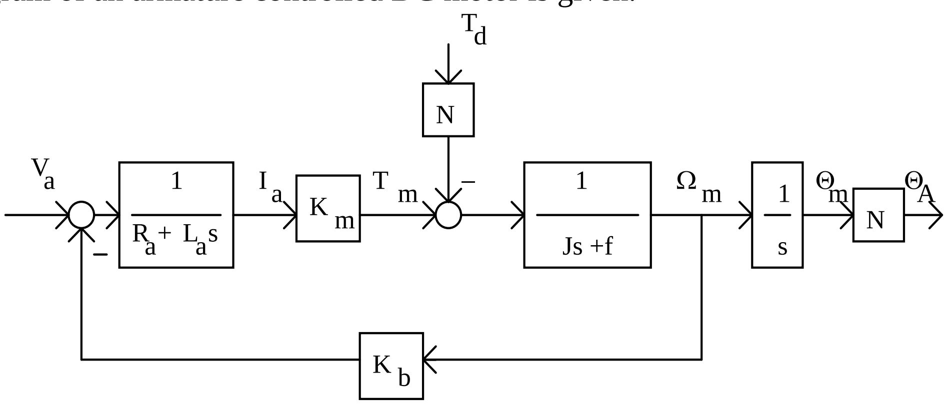


Fig. 1 Block-diagram of an armature controlled DC motor

where Ω_m, Θ_m are rotational speed (rad/s) and position (rad) of a motor, Ω_A is a DC motor position after a gear box (rad). V_a is an armature voltage (V), I_a is an armature current (A), T_m is a torque of a motor (N-m), T_d is a disturbance torque (N-m).

1. Using block-diagram reduction techniques derive transfer functions

$$G_1(s) = \frac{\Theta_A(s)}{V_a(s)} \text{ assuming } T_d = 0 \text{ and } G_2(s) = \frac{\Theta_A(s)}{T_d(s)} \text{ assuming } V_a = 0$$

2. Assuming $K_{\Theta sen} = 1.0\ V/rad$, $R_a = 0.5\,\Omega$, $K_m = 5.0\,N\cdot m/A$, $K_b = 0.1\,V/(rad\cdot s)$, $J = 2.5\,kg\cdot m^2$, $f = 0.2\ N\cdot m/(rad/s)$, $N = 0.1$, $L_a = 0$, a position set point $\Theta_A^* = 1.0\,rad$,

Design a PD compensator and a prefilter (fig. 2),

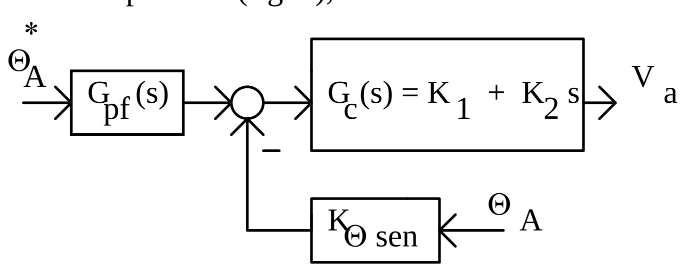


Fig. 2 PD compensator and a prefilter for the DC motor control system

based on a "prototype second order" system concept, i.e. identify the values of the parameters K_1, K_2 and the transfer function $G_{pf}(s)$ of a prefilter to achieve the following performance specifications of the closed loop DC motor control system:
percent overshoot $P.O. = 15\%$, settling time $T_s = 0.75$ sec and steady state error $e_{ss_\Theta} = 0$ (assuming the disturbance torque $T_d(s)$ is equal to zero). Compute a steady state error e_{ss_T} caused by a disurbance $T_d = 120$

<u>Hint.</u> $E_A(s) = \Theta^*_A(s) - \Theta_A(s)$

3. Given a settling time $T_s = 0.75$sec design a PID compensator and a prefilter (fig. 3), using ITAE criterion,

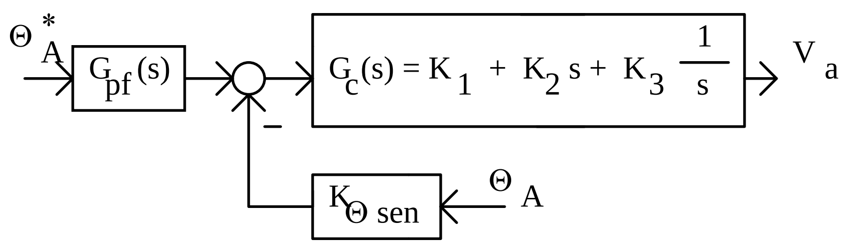


Fig. 3 PID compensator and a prefilter for the DC motor control system

identify the values of parameters K_1, K_2, K_3 and the transfer function $G_{pf}(s)$ of a prefilter to meet the ITAE criterion completely for the closed loop control system. Compute a steady state error e_{ss_Θ} while the disturbance equal to zero and a steady state error e_{ss_T} caused by a disurbance $T_d = 120$

4. Verify stability of the designed closed loop DC-motor position control systems using Routh-Hurwitz criterion of stability

Appendix

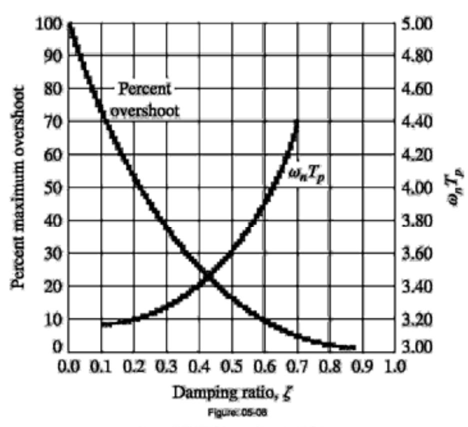
$$t_s = \frac{4}{\xi\omega_n}, \quad t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}, \quad P.O. = 100 \cdot e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

The optimum coefficient based on ITAE criterion for a step input

$s + \omega_n$

$s^2 + 1.4\omega_n s + \omega_n^2$

$s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3$



1. As stem plots, sketch the double-sided amplitude and phase spectra of

a) $x_a(t) = 5\cos(12\pi t - \pi/6)$

b) $x_b(t) = 3\sin(12\pi t) + 4\cos(16\pi t)$

2. An n^{th}-order Butterworth low-pass filter has n left-half-plane poles that lie on a circle in the complex plane.

a) In terms of a filter attribute, what is the radius of this circle?

b) On the above-mentioned circle, what is the angular spacing between the poles (give your answer in radians please)?

3. State Carson's Rule for estimating the bandwidth of a wide-band FM signal. Be sure to define the parameter(s) in this rule.

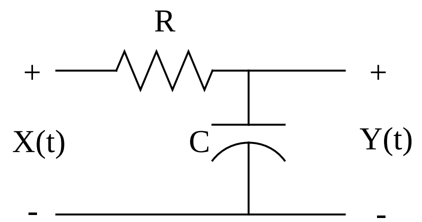
4. Consider the Gaussian random vector $\vec{X} \equiv [x_1 \quad x_2 \quad x_3]^T$ with mean $\vec{\eta} \equiv E[\vec{X}] = \vec{0}$ and covariance matrix

$$\Lambda \equiv E[(\vec{X}-\vec{\eta})(\vec{X}-\vec{\eta})^T] = \begin{bmatrix} 2 & 0 & -3 \\ 0 & 1 & 0 \\ -3 & 0 & 10 \end{bmatrix}.$$

Define the random variables $y_1 = x_1 + 5x_3$, $y_2 = 2x_1 - x_2$. Define the random vector $\vec{Y} \equiv [y_1 \quad y_2]^T$. Write down the density $f(\vec{Y})$.

Hint: A linear transformation of Gaussian random variables is itself Gaussian.

5. Consider the RC lowpass filter shown to the right. Find power density spectrum of output Y if input X is white noise with a power density spectrum of $N_0/2$ watts/Hz.



Preliminary PhD examination in Control Systems, Spring 2010

The angular position of an antenna is controlled by an armature controlled DC motor. A block-diagram of an armature controlled DC motor is given.

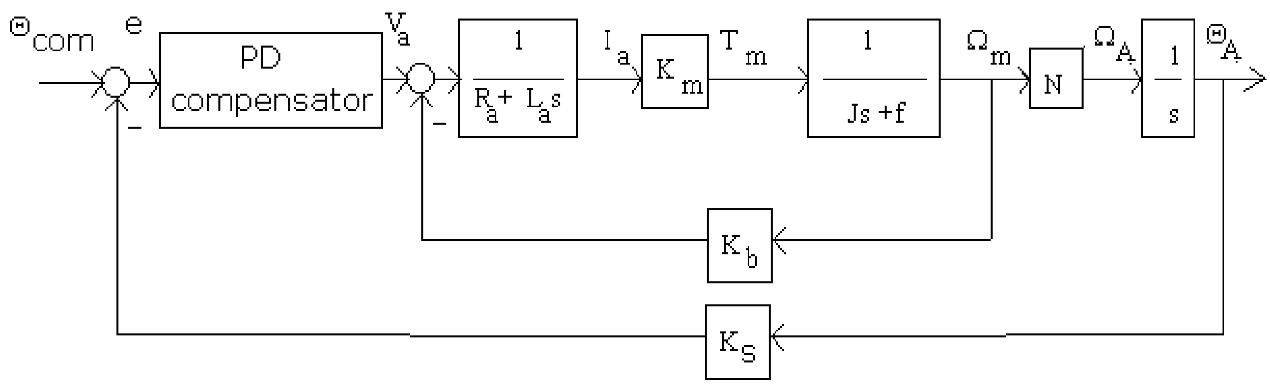


Fig. 1 Block-diagram of an armature controlled DC motor

<u>Designations</u>:

Ω_m - rotational speed (rad/s) of a motor, Ω_A - antenna rotational velocity (rad/s), Θ_A - antenna position (rad), V_a - armature voltage (V), I_a - armature current (A), T_m - torque of a motor (N-m

<u>The following system parameters are given (in SI units):</u>

A gain of a position sensor	$K_s = 1.0\ V/rad$
A gain of a velocity sensor (tachometer)	$K_{s\Omega} = 1.0\ V/rad/s$
An armature resistance of a motor	$R_a = 0.5\ \Omega$
A torque constant of a motor	$K_m = 5.0 N \cdot m/A$
A back-emf constant of a motor	$K_b = 0.1\ V/(rad \cdot s)$
An inertia of a motor's rotor with a load	$J = 2.5\ kg \cdot m^2$
A viscous-friction coefficient of a motor with a load	$f = 0.2\ \ N \cdot m/(rad/s)$
Antenna position set point	$\Theta_{com} = 1.0\ rad$
Antenna initial position	$\Theta_A(0) = 0.0\ rad$
Gear-train ratio	$N = 0.1$

Neglect dynamics of the armature winding assuming $L_a = 0$ at the block-diagram of an armature controlled DC motor (Fig. 1).

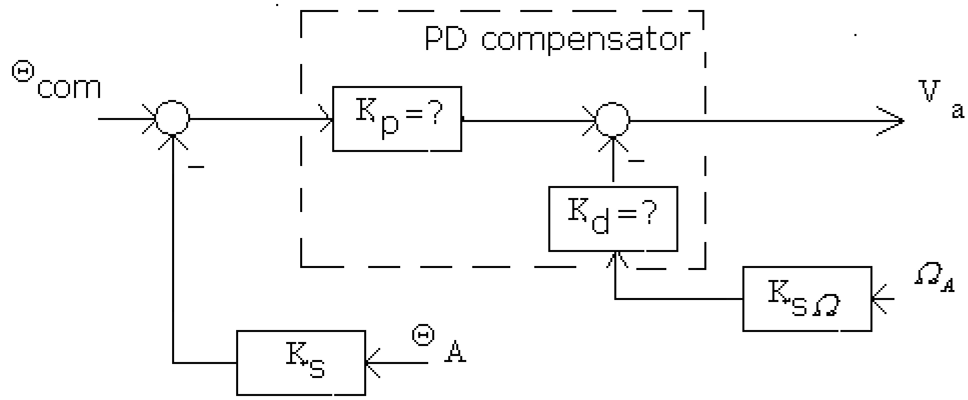


Fig. 2 PD compensator for the antenna position control system

1. Find a transfer function of the plant $G_p(s) = \dfrac{\Theta_A(s)}{V_a(s)}$

2. Design a PD compensator (fig. 2). Based on a "prototype second order" system concept, identify the values of the compensator parameters K_p, K_d to achieve the following performance specifications of the closed loop antenna position control system:
   percent overshoot $P.O. = 15\%$,
   settling time $t_{settling} = 0.5$ sec and
   steady state error $e_{ss_\Theta} = 0$ (assume $\Theta_{com} = 1(t)$).

3. Verify stability of the designed control system using
   a) Routh-Hurwitz criterion of stability,
   b) Nyquist criterion of stability (carefully sketch Nyquist plot),
   c) Bode plot technique (carefully sketch asymptotic Bode plots), identify phase crossover and gain crossover frequencies, phase and gain margins.

4. Based on performance characteristics achieved in task 1 sketch a predicted transient response $\Theta_A(t)$ to a step unit command $\Theta_{com} = 1(t)$.

Appendix

Prototype second order system has a transfer function $\dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

Given a step unit input the specifications of the transient respons of the prototype second order function can be computed as

$$t_{settling} \approx \frac{4}{\xi\omega_n}, \quad PO = 100\% \exp(-\xi\pi / \sqrt{1-\xi^2}), \quad t_{peak} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$